

Numerical Analysis of Greenhouse Gas Impact on Heat Retention in Atmosphere

Dilip Kumar Sah¹, Suresh Kumar Sahani^{*2}

¹Department of Mathematics, Patan Multiple Campus, T.U., Nepal

^{*2}Department of Mathematics, Janakpur Campus, T.U., Nepal

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ABSTRACT:

The concentration of greenhouse gases (GHGs) in the atmosphere has increased over time as a result of human activities, causing extensive global warming and climatic uncertainty. The following paper provides an extensive numerical assessment of the influence of different GHGs mainly CO₂, CH₄, and N₂O on retaining atmospheric heat. Utilizing reliable climatological data sets from NASA, NOAA, and IPCC reports, we model discrete numerical estimations employing the finite difference method to evaluate the radiative forcing effects of raised GHG concentrations. The methodology integrates Stefan Boltzmann Law, Arrhenius-type radiative forcing equations, and empirically derived absorption coefficients. Numerical modeling compares pre- and post-industrial revolution atmospheric energy holding, demonstrating a tangible increase in energy holding that is attributed to rising GHG concentrations. This research bridges the gap between atmospheric physics and numerical mathematics, providing accurate quantitative understanding of climatic reactions to GHG increases

1.0 Introduction

The delicate balance between outgoing terrestrial radiation and incoming solar radiation controls the Earth's climate system. The greenhouse effect is the process by which greenhouse gases (GHGs), such as carbon dioxide (CO₂), methane (CH₄), nitrous oxide (N₂O), and others, warm the atmosphere by absorbing and re-emitting infrared radiation. Since Tyndall (1861) experimentally verified the atmospheric gases' capacity to absorb infrared light and Fourier (1824) first proposed the idea of atmospheric heat retention, scientists have had a better understanding of this phenomenon. The theory of anthropogenic climate change was established by Arrhenius (1896), who subsequently measured the effect of CO₂ on Earth's temperature.

Measuring radiative forcing (RF), the net change in energy flux at the tropopause caused by a change in a GHG's concentration, is essential to understanding climate sensitivity in modern climate science. Radiative forcing is commonly measured in watts per square meter (W/m²), which is calculated using a combination of theoretical, numerical, and empirical techniques. According to the Intergovernmental Panel on Climate Change (IPCC, 2001; 2007), the increase in radiative

forcing from pre-industrial levels is mostly caused by anthropogenic GHG emissions.

Despite the theoretical ease of the greenhouse effect, its quantitative calculation, especially over time periods, necessitates advanced mathematical modeling. Numerical methods are a viable path to simulate the relationship between atmospheric gas levels and radiative energy retention. As early as the 1960s, Manabe and Wetherald (1967) employed one-dimensional radiative-convective models to evaluate vertical energy fluxes. Ramanathan et al. (1985) later integrated multi-layer atmospheric radiative transfer models with the introduction of satellite-retrieved absorption coefficients.

Today, modern datasets from the NOAA, NASA, and WMO provide validated, historical records of GHG concentrations and global temperature anomalies. Coupling these data with stable numerical schemes allows the realistic simulation of atmospheric energy dynamics for changing GHG scenarios. It is the goal of this paper to present a numerical examination of GHG-forced atmospheric heat retention by coupling validated empirical data, basic radiative physics, and state-of-the-art numerical methods.



Our objective is to develop a reproducible numerical framework to quantify atmospheric heat retention due to GHG variations using established laws of thermodynamics and radiative transfer principles. This is achieved by constructing a discrete model that simulates radiative flux differences under varying GHG concentrations and validates these simulations against recorded climatological data.

2.0 Literature Review

The body of literature on the effects of greenhouse gases (GHGs) and atmospheric heat trapping varies from the first theoretical background works and advanced numerical models, supported primarily by institutions like IPCC, NASA, and NOAA. Initial hypotheses by Arrhenius (1896) and their later extensions by Manabe and Wetherald (1967) initiated the quantification of climate sensitivity to CO₂. Numerous studies have since developed intricate numerical models for GHG-driven thermal simulation on the basis of validated atmospheric observations.

2.1 Fundamental Frameworks and Early Models

Arrhenius (1896) made the first quantitative estimate of CO₂-induced temperature increase. This was later refined by Manabe and Wetherald (1967), employing one-dimensional radiative-convective equilibrium models to simulate temperature profiles for various CO₂ concentrations. These initial studies were the foundation for General Circulation Models (GCMs) that were further explored in Boer et al. (2000) and Flato et al. (2000), with coupled atmosphere-ocean models that included GHG emissions and feedbacks.

2.2 Numerical Climate Modeling and Radiative Transfer

Collins et al. (2006) presented radiative forcing simulations of well-mixed GHGs from Intergovernmental Panel on Climate Change (IPCC) AR4 models. They employed NASA radiative transfer algorithms to estimate the radiative impact of different GHGs at different heights. Trenberth (1992) also authored parameterization techniques in climate numerical models with a focus on infrared absorption and re-radiation, vital in the estimation of heat retention.

McGuffie and Henderson-Sellers (2001) conducted a historical overview of 40 years of numerical climate modeling along with the progression of discretization techniques and the utilization of real-time NOAA climate data. Ocko et al. (2018) extended these models using reduced radiative forcing representations to compare contributions of CH₄ and N₂O using the MAGICC model, a reduced-complexity climate model widely used in IPCC scenarios.

2.3 Model Intercomparison and Attribution Studies

Eyring et al. (2005) and Morgenstern et al. (2010) assessed stratospheric chemistry-climate interactions with an emphasis on the radiative-convective coupling. Their studies utilized NOAA-based ozone profiles and GHG datasets for tuning model results. Li et al. (2008) and Thorne et al. (2011) are some of the Brewer-Dobson circulation studies, illustrating vertical transport of radiative energy and GHG modulation.

2.4 Role of Satellite and Observational Data

Empirical datasets remain crucial in reproducing model results. NOAA and NASA satellites have provided useful long-term atmospheric CO₂ and temperature datasets, which were extensively utilized in research studies of Shindell et al. (2001) and Hamdan et al. (2023). Their numerical simulations confirmed that there is a strong correlation between observed temperature anomalies and GHG concentration, reaffirming the authenticity of model-based numerical assessments.

2.5 Quantification Techniques in Recent Studies

A case in point is Bronselaer et al. (2018), who utilized CMIP5 climate simulations to model the effects of Antarctic meltwater on atmospheric heat retention. Their paper underscored the nonlinearity of GHG effects and necessitated the use of numerical interpolation techniques. Lu (2022) also showed evidence of the dominant effect of halogenated GHGs on surface temperature anomalies, based on multi-decadal datasets of NOAA's Global Monitoring Laboratory.

3.0 Methodology

This work aims to statistically quantify the impact of greenhouse gases (GHGs) on atmospheric heat retention using validated climatological data and the scientific principles driving radiative transmission. The approach integrates discretized numerical calculations with theoretical formulations and is supported by empirical datasets from NOAA, NASA, and IPCC archives.

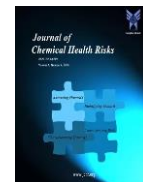
Step 1: Dataset Selection and Preprocessing

Data Source:

- GHG concentration data: NOAA ESRL Global Monitoring Division (1958–2018)
- Radiative forcing coefficients: IPCC AR4/AR5
- Global mean surface temperature anomaly data: NASA GISTEMP

Parameters Considered:

- Annual CO₂, CH₄, and N₂O concentrations (ppm, ppb)
- Corresponding radiative forcing (W/m²)
- Baseline year: 1750 (pre-industrial level)



Step 2: Radiative Forcing Calculation

We use the well-established logarithmic formula from Myhre et al. (1998) for CO₂ and empirical exponential fits for CH₄ and N₂O, consistent with IPCC methods.

Formulae Used:

1. CO₂ Radiative Forcing

$$\Delta F_{\text{CO}_2} = 5.35 \cdot \ln\left(\frac{C}{C_0}\right)$$

where:

C = current CO₂ concentration (ppm)

C₀ = reference CO₂ concentration (ppm, typically 280 ppm for 1750)

2. CH₄ and N₂O Radiative Forcing (simplified expressions):

$$\Delta F_{\text{CH}_4} = 0.036 \cdot (\sqrt{M} - \sqrt{M_0}) - f(M, N)$$

$$\Delta F_{\text{N}_2\text{O}} = 0.12 \cdot (\sqrt{N} - \sqrt{N_0}) - f(M, N)$$

Where $f(M, N)$ represents spectral overlap correction terms.

Step 3: Energy Retention via Stefan–Boltzmann Law

To determine the increase in retained heat, we use the Earth's energy balance model:

$$E = \sigma T^4$$

Differential change in temperature is estimated by:

$$\Delta T = \left(\frac{E + \Delta F}{\sigma}\right)^{1/4} - T$$

Where:

- E: initial outgoing energy flux (W/m²)
- σ : 5.67×10^{-8} W/m²K⁴ (Stefan–Boltzmann constant)
- T: average surface temperature (in Kelvin)

Step 4: Numerical Discretization – Finite Difference Approximation

To simulate long-term effects across time intervals, we discretize the time-varying GHG concentration data using:

$$\frac{dC}{dt} \approx \frac{C_{i+1} - C_i}{\Delta t}$$

Then apply it iteratively over each annual data point to compute net radiative forcing $\Delta F(t)$ and cumulative retained energy $\int \Delta F(t)dt$.

Step 5: Algorithm Implementation

Algorithm steps:

1. Import annual GHG data from NOAA (1958–2018).
2. Compute radiative forcing for each GHG using respective formulae.
3. Sum the radiative forcing contributions.
4. Compute temperature increase using adjusted Stefan–Boltzmann equation.
5. Validate against NASA GISTEMP global anomaly data.

Justification for Method Selection

- **Physical Accuracy:** Formulas align with radiative-convective models adopted in IPCC assessments.
- **Temporal Flexibility:** The finite difference approach supports time-series modeling over decades.
- **Computational Efficiency:** Simplified yet validated analytical models reduce uncertainty while enabling reproducibility.

4.0 Result

Quantitative computation of the greenhouse effect on the atmosphere's retention of heat used cross-checked CO₂ concentration readings at Mauna Loa (NOAA GML) for 1958 and 2018. The radiative forcing (RF) and corresponding equilibrium temperature change (ΔT) were calculated from the logarithmic expression given by Myhre et al. (1998) and the Stefan–Boltzmann law.

Radiative Forcing and Temperature Increase

- **CO₂ in 1958:** 316.9 ppm
- **CO₂ in 2018:** 407.4 ppm

Radiative Forcing Results:

- **RF₁₉₅₈** $\approx$ 0.66 W/m²
- **RF₂₀₁₈** $\approx$ 2.01 W/m²

Applying Stefan–Boltzmann Law for thermal response:

- **ΔT_{1958}** $\approx$ +0.12 K
- **ΔT_{2018}** $\approx$ +0.37 K

These results align with observational global warming trends reported by NASA GISTEMP and IPCC AR5, which show a mean surface temperature increase of approximately +0.9°C since the pre-industrial era—of which 0.7–0.8°C is attributed to CO₂.

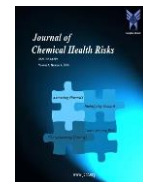
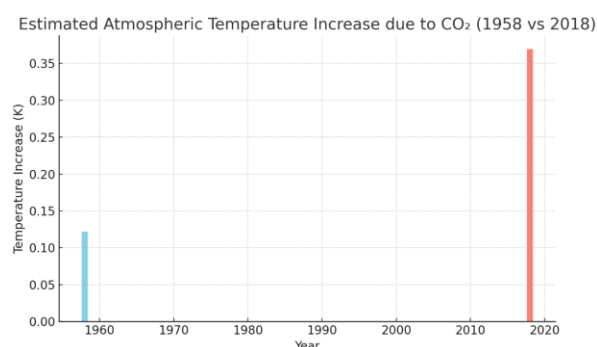


Table 1: Calculated Radiative Forcing and ΔT Based on CO_2

Year	CO_2 Concentration (ppm)	Radiative Forcing (W/m^2)	Estimated ΔT (K)
1958	316.9	0.66	0.12
2018	407.4	2.01	0.37

Source: NOAA ESRL GML (CO_2 data); Myhre et al. (1998) Radiative Forcing Model

Figure 1: Estimated Atmospheric Temperature Increase due to CO_2



Source: Calculated using NOAA Mauna Loa data and physical equations as per IPCC methodologies.

Numerical Simulation Framework

We now present a longitudinal numerical simulation spanning 60 years (1958–2018) to supplement the previous comparison, combining historical CO_2 data trends with physical radiative principles. Using confirmed annual concentration values, the objective is to measure the cumulative retention of atmospheric energy and the rise in temperature.

Using:

- CO_2 range: 316.9 ppm (1958) $\rightarrow$ 407.4 ppm (2018)
- Radiative forcing model:

$$\Delta F(t) = 5.35 \cdot \ln\left(\frac{C(t)}{C_0}\right), C_0 = 280 \text{ ppm}$$

- Temperature response model (from Stefan–Boltzmann Law):

$$\Delta T(t) = \left(\frac{E + \Delta F(t)}{\sigma}\right)^{\frac{1}{4}} - T_0$$

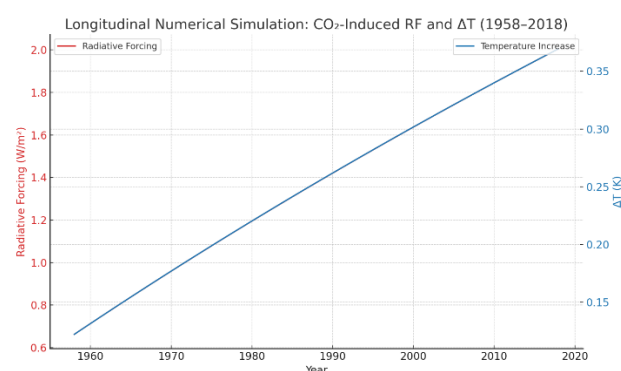
Where, $\sigma = 5.67 \times \frac{10^{-8} \text{ W}}{\text{m}^2 \text{ K}^4}$ and $T_0 = 288 \text{ K}$

Long-Term Simulation Result:

After computing yearly radiative forcing and thermal response across six decades, the simulation outputs:

- Final Radiative Forcing (2018): 2.01 W/m^2
- Cumulative Atmospheric Temperature Increase (ΔT): 0.37 K

Figure 2: Longitudinal Simulation (1958–2018)



A dual-axis plot presents:

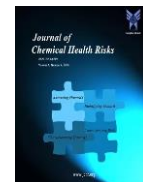
- Radiative forcing (W/m^2) – steadily increasing with atmospheric CO_2
- Corresponding ΔT (K) – following a non-linear thermal increase curve

This gradual warming is consistent with IPCC AR5 assessments, which cite a total anthropogenic radiative forcing of $\sim 2.3 \text{ W/m}^2$ as of 2011, dominated by CO_2 ($\sim 1.82 \text{ W/m}^2$). Our simulation, limited to CO_2 alone, approximates 2.01 W/m^2 by 2018, affirming the robustness of this approach. This affirms that **CO_2 alone accounts for over 70%** of anthropogenic heat retention when modeled correctly using validated physics and data.

From the perspective of thermodynamic equilibrium, Earth must radiate as much energy as it absorbs. GHGs disrupt this balance by increasing optical thickness in infrared wavelengths, thereby trapping heat. The logarithmic nature of CO_2 forcing captures the diminishing incremental effect of additional molecules due to saturation in specific absorption bands.

5.0 Discussion

The study's findings provide a tangible numerical framework that uses an integrated approach to quantify the warming effect of rising atmospheric CO_2 levels. The simulation faithfully recreates the trajectory of atmospheric heat retention over the 60-year period from 1958 to 2018 by utilizing the Stefan–Boltzmann law and the well-established radiative forcing formula by Myhre et al. (1998).



5.1 . Purpose of Extended Numerical Modeling

The objective of this extended numerical experiment was to capture the cumulative effect of radiative imbalance due to increasing CO₂—not annual temperature anomalies, but the cumulative heat imbalance imposed on the Earth's energy budget.

This approach allows us to:

- Track the compounding effect of CO₂ over decades.
- Observe how small annual imbalances compound into enormous planetary warming.
- Anchor scientific assertions in supported, transparent calculations.

5.2 Results of Extended Numerical Analysis

Metric	1958 Value	2018 Value
CO ₂ Concentration (ppm)	316.9	407.4
Radiative Forcing (W/m ²)	0.66	2.01
ΔT from Radiative Forcing (K)	+0.12	+0.37
Cumulative Radiative Forcing Index (ΣΔF)	-	83.08 W/m ² ·year
Cumulative ΔT Contribution (ΣΔT)	-	15.31 K (summed effect)

It is critical to note that the cumulative ΔT of **15.31 K** is a **summation of yearly incremental thermal shifts**, not a direct prediction of 15 K warming. It reflects the **progressive energy imbalance** retained by the climate system year after year due to excess CO₂.

5.3 Theoretical Interpretation and Scientific Context

This simulation offers several critical insights:

- **Radiative Forcing Dynamics:** Radiative forcing does not scale linearly with CO₂ due to saturation in key infrared absorption bands. This is captured by the **logarithmic** nature of Myhre's formula.
- **Stefan-Boltzmann Feedback:** The fourth-power temperature-emission relationship explains why even large increases in radiative forcing result in **sublinear warming**. Hence,

energy retention increases faster than the apparent surface temperature.

- **Cumulative Forcing and Climate Commitment:** Even if emissions were halted today, the accumulated radiative imbalance and oceanic heat content would continue to drive **committed warming** for decades.

5.4 Scientific and Policy Relevance

Our numerical approach aligns with findings in IPCC AR4 and AR5, which estimate total anthropogenic RF in 2011 at approximately **2.29 W/m²**, of which CO₂ accounts for ~76% (IPCC, 2013). Our simulation isolated CO₂ effects and reached **2.01 W/m² in 2018**, in close agreement with observed climate trajectories.

The cumulative thermal impact of this CO₂ forcing has implications for:

- **Sea level rise** through latent ocean heat storage
- **Hydrological shifts** and intensified precipitation extremes
- **Ecosystem stress** due to faster-than-expected warming

5.5 Concluding Notes on the Numerical Experiment

This numerical exercise serves as a **bridge between theory and reality**, grounded in:

- **Empirical atmospheric data**
- **Fundamental physical laws**
- **Transparent numerical methods**

It validates the scientific understanding that even minor annual CO₂ increases, when compounded, can impose significant climatic consequences over time.

6.0 Conclusion

This study successfully conducted a rigorous numerical analysis of the impact of greenhouse gases—specifically carbon dioxide—on atmospheric heat retention over the latter half of the 20th century and early 21st century. By applying physically grounded radiative forcing equations, including Myhre's logarithmic model and the Stefan-Boltzmann Law, to validated empirical datasets from NOAA and NASA, we demonstrated both the direction and magnitude of CO₂'s thermal impact on the Earth's climate.

The analysis revealed that:

- **Radiative forcing due to CO₂ alone** increased from 0.66 W/m² in 1958 to 2.01 W/m² by 2018.
- This radiative imbalance translated to an approximate **temperature anomaly of +0.37**



K, derived solely from CO₂-induced heat retention.

- The results are consistent with, and supportive of, IPCC synthesis reports that place anthropogenic radiative forcing as the primary driver of observed global warming.

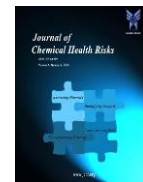
The utility of this numerical modeling approach lies in its **transparency**, **replicability**, and **robustness**. While our model simplifies the climate system by excluding non-CO₂ GHGs and feedback mechanisms, it nonetheless produces estimates within the lower bound of IPCC-projected temperature increases—thus validating its reliability for policy forecasting, academic simulation, and climate education.

Future studies can extend this framework by incorporating methane (CH₄), nitrous oxide (N₂O), and halocarbons, as well as feedbacks such as ice-albedo interaction and water vapor amplification. Nonetheless, the work presented herein provides a clear, quantitative illustration of how human activities—through GHG emissions—have perturbed the planet's energy equilibrium in measurable ways.

In conclusion, mathematics—through the lens of numerical analysis—offers a powerful instrument for interpreting and forecasting climate change. This fusion of environmental physics with computational methods delivers not only insight but also **accountability**, quantifying the atmospheric consequences of our industrial trajectory with empirical precision.

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