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Theoretical Concepts for the Morphology Study of Insects Using Topological Descriptors

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KEYWORDS

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ABSTRACT:
Since they aid in the pollination of sexually reproducing plants and boost agricultural yields for human consumption, insects are a crucial component of the ecosystem's fauna. The current work's primary goal was to determine the order of insects by computing a number of topological characteristics (TDs). These indices were first used to establish the order of insects by analysing the wings of diverse insect species. A wing graph has been created and utilised worldwide for the first time. The wings of Drosophila, Cicada, Apis, and Musca species were amputated and utilised to make durable slides. Several vertices (points where venation overlaps) and edges were used to evaluate the venation systems of various wings (part of vein between two vertices). Wing graphs were made using photos of the wings, and the researched indices were identified. This study is significant for future research because insects can be identified and the order of insects may be established using these indices. For a certain insect order, the values of a few topological descriptors were calculated. This work will be crucial in the future since using these indices enables the distinction of insects and the identification of insect order. The first and second Zagreb indices, the F-index, the reformulated and modified Zagreb indices, the symmetric division index, the inverse sum index, the harmonic index, and the augmented Zagreb index are all found in this work. The results on these topics relate to both theoretical and empirical data on their symmetry properties.

1. Introduction

Hexapod invertebrates known as insects are members of the Insecta class of the Arthropoda phylum. The name "insect" originates from the Latin word "insectum." All members of the insecta class have been found to have three pairs of legs, compound eyes, one pair of antennae, a chitinous exoskeleton, and bodies that are separated into the head, thorax, and abdomen. Over 90% of all living forms on Earth are insects [5]. By assisting in the nutrient cycle, seed dispersal, plant pollination, and preservation of soil structure, insects created the biological underpinning for all land-dwelling creatures [7]. Because of this, insects are crucial to human welfare. In the scientific community, there are many methods for classifying insects using

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entomological keys and research literature; however, the current study is the first to apply a novel idea for classifying insect orders by computing graph polynomials and various TDs with the aid of the venation vertices of the wings. When correlating topological research to the chemical characteristics of various compounds or the therapeutic effects of medications, graph polynomials represent novel findings of topological indices [10]. In the current study, it was employed to distinguish across insect orders since the venation pattern and vertices on the wings of various insect orders varied. For the first time, this concept has been used to identify insect orders. The behavior of these polynomials was illustrated using images of several types of wings.

2. Material and Methods

Using a hand sweeping net, four distinct varieties of insects (Drosophila melanogaster, Cicada spp, Apis spp, and Musca spp) were caught and stored in insect killing bottles. These four insects belonged to four separate families and three different orders of the insecta class, namely the Diptera (Drosophila), Hemiptera (Cicada spp), Hymenoptera (Apis spp), and Diptera (Musca spp). Entomological boxes made of wood were used to store insects.

All caught insects had their wings delicately amputated using forceps, and then each wing was placed on a glass microscopic slide with a few drops of Canada balsam and sealed with a glass cover slip for long-term preservation. Stereo zoom microscope images of several insect wings under study. After printing out the images of the wings, the graphs of the veins were created. On networks of wings, various topological vertices were labeled as 1^0, 2^0, 3^0, and 4^0 vertices, see Figures 1 to 4. The segment of a vein in the wing that crosses over or ends is termed a vertex, and the edge is the portion of the vein that connects two close vertices. Plotting wing graphs included joining several vertices and edges, see Figures 1 to 4. A vertex is referred to as having one, two, three, or four additional vertices attached to it, correspondingly. A vertex which is connected from one, two, three or four other vertices is called 1^0, 2^0, 3^0, and 4^0, respectively.

It was discovered that the patterns of vertices in the venation system of the wings of various insects varied and were differentiated. Varied insect species' TDs of their wing venation were evaluated, and it was discovered that insects from various orders of insecta have different values for various TDs. While insects from various orders, such as Hymenoptera, Diptera, and Hemiptera, exhibited significantly different values of estimated TDs, insects from the same order but belonging to distinct families displayed extremely similar TDs values.

Numerous fields, including biology, mathematics, bioinformatics, informatics, and others, have used topological descriptors in their research. A topological descriptor is a function that characterizes the topology of the graph. Most important and commonly known indices are degree-based descriptors. These TDs are actually the numerical values that correlate the structure with various physical properties reactivity, and biological activities.

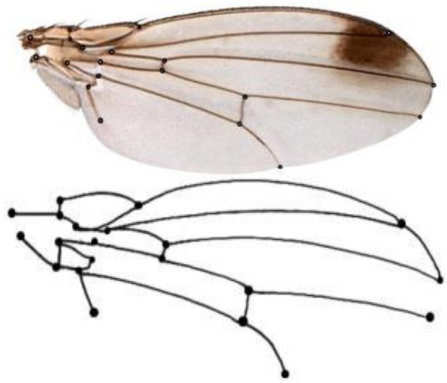

Figure 1. Drosophila spp and its wing graph**.**

Figure 2. Cicada spp and its wing graph.

Figure 3. Apis spp and its wing graph.

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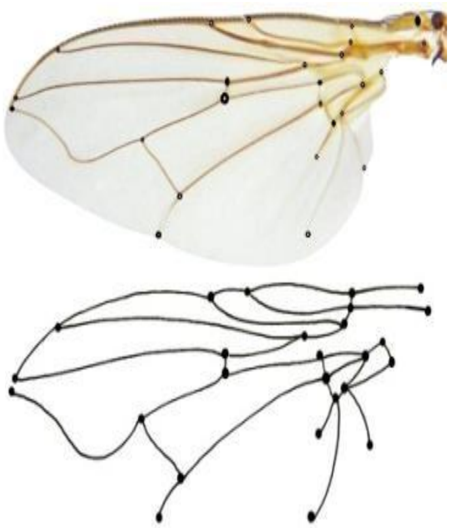

Figure 4. Musca spp and its wing graph.

Let $G = (V, E)$ denote a graph with set of vertices V and two element subsets of V, known as the edges forming E. The degree of a vertex r is the number of vertices at distance one denoted as ζ_r. In the literature, a number of graph polynomials were introduced, and some of them proved to be beneficial in Biology and Chemistry. Graph polynomials are functions of isomorphism-invariant graphs. Usually, they are polynomials with integer coefficients in one or two variables. Various applications of degree-based TDs were studied by many researchers in [2, 3, 8, 9, 12, 13, 16, 17, 23-34]. Let us use the following notation for the rest of the paper.

$V_i = \{s \in V(G) | \zeta_s = i\}$ an $\zeta|V_i| = n_i$

$m_{ij} = |E_{ij}|$, where $E_{ij} = \{rs \in E(G) | \zeta_r = i, \zeta_s = j\}$

$\bar{m}_{ij} = |\bar{E}_{ij}|$, where $\bar{E}_{ij} = \{rs \notin E(G) | \zeta_r = i, \zeta_s = j\}$

The M-polynomial [4] of graph G is defined as

$$M(G; r, s) = \sum_{i \le j} m_{ij}(G) r^i s^j,$$

where $m_{ij}(G), i, j \ge 1$ be the number of edges $rs \in E(G)$ such that $\{(\zeta_r, \zeta_s) = (i, j)\}$

Now we concentrate for non adjacent pair of vertices and define $\bar{M}$- Polynomial [22] like M – polynomial as follows;

$$\bar{M}(G; r, s) = \sum_{i \le j} \bar{m}_{ij}(G) r^i s^j,$$

where $\bar{m}_{ij}(G), i, j \ge 1$be the number of edges $rs \notin E(G)$ such that $\{(\zeta_r, \zeta_s) = (i, j)\}$

Let $\xi(s)$ represent the neighbourhood degree of s in the graph G, that is,

$\xi(s) = \sum_{u \in N_G(r)} \zeta(r)$, where $N_G(r)$ being the set of adjacent vertices of u.

Let $\chi_{i,j}(G); i, j \ge 1$ be the number of edges $e = rs$ of G such that $\{\xi(r), \xi(s)\} = \{i, j\}$ then the NM-Polynomial [19, 20, 21] of graph G is defined as

$$NM(G; r, s) = \sum_{i \le j} \chi_{i,j}(G) r^i s^j.$$

3. Formulation of some TDs from polynomials

Some degree-based, some neighbourhood degree-based TDs and their association with $\bar{\mathrm{M}}$ -Polynomial and NM-Polynomial of a graph G respectively are given below in the Table 1 and 2 of form

$\mathfrak{D}_r = r\frac{\partial(f(r,s))}{\partial r}, \mathfrak{D}_s = s\frac{\partial(f(r,s))}{\partial s}, \mathfrak{S}_r = \int_0^r \frac{f(t,s)}{t} dt, \mathfrak{S}_r = \int_0^s \frac{f(r,t)}{t} dt, \mathfrak{J}(f(r,s)) = f(r,r)$ and

$$\mathfrak{Q}_k(f(r,s)) = r^k f(r,s).$$

Degree – based TDs	**Mathematical Expression**	**Derivative from $f(x,y)$**
First Zagreb coindex $\bar{M}_1$	$\sum_{rs \notin E(G)} (\zeta(r) + \zeta(s))$	$(\mathfrak{D}_r + \mathfrak{D}_s)(f(r,s))\|r = s = 1$
Second Zagreb coindex $\bar{M}_2$	$\sum_{rs \in E(G)} (\zeta(r)\zeta(s))$	$(\mathfrak{D}_r \mathfrak{D}_s)(f(r,s))\|r = s = 1$
F-coindex $\bar{F}$	$\sum_{rs \in E(G)} (\zeta^2(r) + \zeta^2(s))$	$(\mathfrak{D}_r^2 + \mathfrak{D}_s^2)\ (f(r,s))r = s = 1$
Second modified Zagreb coindex $m\bar{M}_2$	$\sum_{rs \in E(G)} \frac{1}{\zeta(r)\zeta(s)}$	$(\mathfrak{S}_r \mathfrak{S}_s)(f(r,s))\|r = s = 1$
Symmetric deg division coindex $\overline{SDD}$	$\sum_{rs \in E(G)} \frac{\zeta^2(r) + \zeta^2(s)}{\zeta(r)\zeta(s)}$	$(\mathfrak{D}_r \mathfrak{S}_s + \mathfrak{S}_r \mathfrak{D}_s)(f(r,s))\|r = s = 1$

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Harmonic coindex $\overline{H}$	$\sum_{rs\in E(G)} \frac{2}{\zeta(r)+\zeta(s)}$	$(2\mathfrak{S}_r\mathfrak{F})(f(r,s))\|r=1$
Inverse sum index coindex $\overline{ISI}$	$\sum_{rs\in E(G)} \frac{\zeta(r)\zeta(s)}{\zeta(r)+\zeta(s)}$	$(\mathfrak{S}_r\mathfrak{J}\mathfrak{D}_r\mathfrak{D}_s)(f(r,s))\|r=1$
Augmented Zagreb coindex $\bar{A}$	$\sum_{rs\in E(G)} \left(\frac{\zeta(r)\zeta(s)}{\zeta(r)\zeta(s)-2}\right)^3$	$(\mathfrak{S}_r^3\mathfrak{Q}_{-2}\,\mathfrak{J}\,\mathfrak{D}_r^3\mathfrak{D}_s^3)(f(r,s))\|r=1,$

Table 1. Description of some degree-based descriptors.

Neighbourhood degree – based	**Mathematical Expression**	**Derivative from $NM\ (G)$**
Neighbourhood first Zagreb index NM_1	$\sum_{rs\in E(G)} (\xi(r)+\xi(s))$	$(\mathfrak{D}_r+\mathfrak{D}_s)(NM(G))\|r=s=1$
Neighbourhood second Zagreb index NM_2	$\sum_{rs\in E(G)} (\xi(r)\xi(s))$	$(\mathfrak{D}_r\mathfrak{D}_s)(NM(G))\|r=s=1$
Neighbourhood F-index NF	$\sum_{rs\in E(G)} (\xi^2(r)+\xi^2(s))$	$(\mathfrak{D}_r^2+\mathfrak{D}_s^2)(NM(G))\|r=s=1$
Neighbourhood Second modified Zagreb index NM^*	$\sum_{rs\in E(G)} \frac{1}{\xi(r)\xi(s)}$	$(\mathfrak{S}_r\mathfrak{S}_s)(NM(G))\|r=s=1$
The third ND_lindex ND_3	$\sum_{rs\in E(G)} (\xi(r)\xi(s))(\xi(r)+\xi(s))$	$(\mathfrak{D}_r\mathfrak{D}_s(\mathfrak{D}_r+\mathfrak{D}_s))(NM(G))\|r=s=1$
Neighbourhood symmetric deg division index NSD	$\sum_{rs\in E(G)} \frac{\xi^2(r)+\xi^2(s)}{\xi(r)\xi(s)}$	$(\mathfrak{D}_r\mathfrak{S}_y+\mathfrak{S}_r\mathfrak{D}_s)(NM(G))\|r=s=1$
Neighbourhood harmonic index NH	$\sum_{rs\in E(G)} \frac{2}{\xi(r)+\xi(s)}$	$(2\mathfrak{S}_r\mathfrak{J})(NM(G))\|r=1$
Neighbourhood inverse sum index $NISI$	$\sum_{rs\in E(G)} \frac{\xi(r)\xi(s)}{\xi(r)+\xi(s)}$	$(\mathfrak{S}_r\mathfrak{J}\mathfrak{D}_r\mathfrak{D}_s)(NM(G))\|r=1$
Sanskruti index S	$\sum_{rs\in E(G)} \left(\frac{\xi(r)\xi(s)}{\xi(r)\xi(s)-2}\right)^3$	$(\mathfrak{S}_r^3\mathfrak{Q}_{-2}\,\mathfrak{J}\,\mathfrak{D}_r^3\mathfrak{D}_s^3)(NM(G))\|r=1,$

Table 2. Description of some neighbourhood degree-based descriptors.

4. Graph polynomials for Drosophila Species

In this section, we find the several TDs based on degree and neighbourhood degree of a wing graph of Drosophila. The proof of the following lemma was presented by Berhe and Wang [18].

Lemma 1. [18] For a connected graph G of n vertices, we have $\overline{m}_{ij} = |\bar{E}_{ij}| = \frac{(n_i-1)n_i}{2} - m_{ij}$ whenever $i = j$ and $\overline{m}_{ij} = |\bar{E}_{ij}| = n_i n_j - m_{ij}$ whenever $i < j$.

Theorem 1. Let G_1 be a wing graph of Drosophila. Then

(i) The $\overline{M}$-polynomial of $G_1$is $\overline{M}(G_1;r,s) = 10rs^3 + 33r^2s^3 + 64r^3s^3$

(ii) The NM-polynomial of $G_1$is $NM(G_1;r,s) = r^3s^6 + r^6s^6 + r^6s^9 + 4r^6s^8 + 5r^8s^9 + r^6s^8 + r^4s^9 + r^7s^9 + 2r^4s^8 + 2r^3s^8 + r^3s^4 + 2r^8s^8 + r^6s^7 + r^7s^7 + r^9s^9$.

Proof. From the Figure 1, it is easy to calculate that $|V(G_1)| = 21$ and $|E(G_1)| = 25$. The vertex set can be partition into three subsets, namely, $n_1 = |s_1| = 5, n_2 =$

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$|s_2| = 3$ *and* $n_3 = |s_3| = 13$. Also, the edge set of G_1 may be categorize into three categories based on the degree of vertices

$$E_{13} = \{rs \in E(G_1) | \zeta(r) = 1, \zeta(s) = 3\}$$
$$E_{23} = \{rs \in E(G_1) | \zeta(r) = 2, \zeta(s) = 3\}$$
$$E_{33} = \{rs \in E(G_1) | \zeta(r) = \zeta(s) = 3\}$$

such that $m_{13} = |E_{13}| = 5, m_{23} = |E_{23}| = 6$ and $m_{33} = |E_{33}| = 14$ Using Lemma 1, we have

$$\overline{m_{13}} = n_1 n_3 - m_{13} = 5(13) - 5 = 10$$
$$\overline{m_{23}} = n_2 n_3 - m_{23} = 3(13) - 6 = 33$$
$$\overline{m_{33}} = \frac{n_3(n_3-1)}{2} - m_{33} = \frac{13(12)}{2} - 14 = 64$$

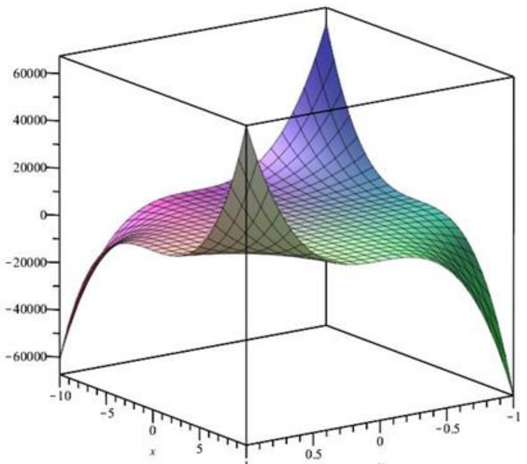
By definition of $\overline{\mathrm{M}}$ - Polynomial,

$$\overline{M}(G_i; r, s) = \sum_{i \le j} \overline{m_{ij}}\,(G) r^i s^j$$
$$= \sum_{1\le 3} \overline{m_{13}}\, rs^3 + \sum_{2\le 3} \overline{m_{23}}\, r^2 s^3 + \sum_{3\le 3} \overline{m_{33}}\, r^3 s^3$$
$$= 10rs^3 + 33r^2s^3 + 64r^3s^3.$$

$\chi_{a,b}$ denote the set of all edges with neighbourhood degree sum of end vertices a and b. From the structure of Figure 1, we have $|\chi_{3,6}| = 1, |\chi_{3,6}| = 1, |\chi_{6,6}| = 1, |\chi_{6,9}| = 1,$
$|\chi_{6,8}| = 4, |\chi_{8,9}| = 5, |\chi_{7,8}| = 1, |\chi_{4,9}| = 1, |\chi_{7,9}| = 1, |\chi_{4,8}| = 2, |\chi_{3,8}| = 2,$
$|\chi_{3,4}| = 1, |\chi_{8,8}| = 2, |\chi_{6,7}| = 1.$

Thus from the definition of NM – Polynomial, we have

$$NM\,(G_1; r, s) = \sum_{i \le j} \chi_{ab} r^a s^b$$
$$= \chi_{3,6}r^3s^6 + \chi_{6,6}r^6s^6 + \chi_{6,9}r^6s^9 + \chi_{6,8}r^6s^8 + \chi_{8,9}r^8s^9 + \chi_{7,8}r^7s^8 + \chi_{4,9}r^4s^9 + \chi_{7,9}r^7s^9 + \chi_{4,8}r^4s^8 + \chi_{3,8}r^3s^8 + \chi_{3,4}r^3s^4 + \chi_{8,8}r^8s^8 + \chi_{6,7}r^6s^7 + \chi_{7,7}r^7s^7 + \chi_{9,9}r^9s^9$$
$$= r^3s^6 + r^6s^6 + r^6s^9 + 4r^6s^8 + 5r^8s^9 + r^6s^8 + r^4s^9 + r^7s^9 + 2r^4s^8 + 2r^3s^8 + r^3s^4 + 2r^8s^8 + r^6s^7 + r^7s^7 + r^9s^9.$$



Hence the result.

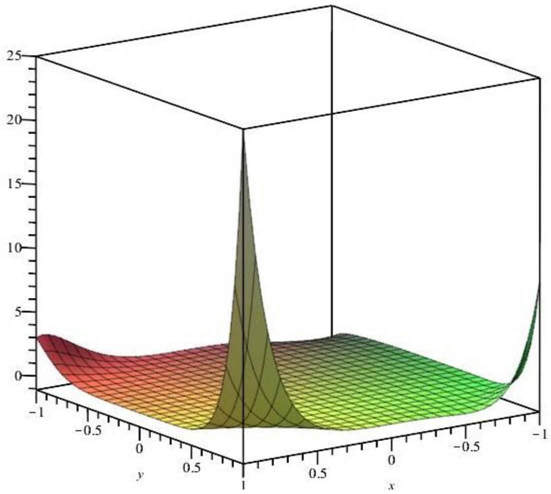


Figure 5. 3D-plots for $\overline{\mathrm{M}}$ -polynomial and NM-polynomial of wing graph of Drosophila

The 3D- Plots of $\overline{\mathrm{M}}$ -polynomial and NM-polynomial of wing graph of Drosophila species showed different patterns in Figure 5. Now we present some degree-based and neighbourhood degree- based TDs of wing graph for Drosophila using $\overline{\mathrm{M}}$ -polynomial and NM-polynomial. From Table 1, we obtain the following;

$$\overline{M}_1(G_1) = (\mathfrak{D}_r + \mathfrak{D}_s)(f(r,s))(1,1)$$
$$= (40rs^3 + 165r^2s^3 + 384r^3s^3)(1,1) = 589$$
$$\overline{M}_2(G_1) = \mathfrak{D}_r\mathfrak{D}_s(f(r,s))(1,1)$$
$$= (30rs^3 + 198r^2s^3 + 576r^3s^3)(1,1) = 804$$
$$m\overline{M}_2(G_1) = \mathfrak{S}_r\mathfrak{S}_s(f(r,s))(1,1)$$
$$= \left(\frac{10}{3}rs^3 + \frac{33}{6}r^2s^3 + \frac{64}{9}r^3s^3\right)(1,1) = 15.94$$
$$\overline{RZ}(G) = (\mathfrak{D}_r\mathfrak{D}_s)(\mathfrak{D}_r + \mathfrak{D}_s)\big(f(r,s)\big)(1,1)$$
$$= (120rs^3 + 990r^2s^3 + 3456r^3s^3)(1,1) = 4566$$
$$\overline{F}(G_1) = (\mathfrak{D}_r^2 + \mathfrak{D}_s^2)(f(r,s))(1,1)$$
$$= (100rs^3 + 429r^2s^3 + 1152r^3ys^3)(1,1) = 1681$$
$$\overline{SDD}(G_1) = (\mathfrak{D}_r\mathfrak{S}_s + \mathfrak{S}_r\mathfrak{D}_s)(f(r,s))(1,1)$$
$$= \left(\left(\frac{10}{3} + 30\right)rs^3 + \left(\frac{66}{3} + \frac{99}{2}\right)r^2s^3 + \left(\frac{192}{3} + \frac{192}{3}\right)r^3s^3\right)(1,1)$$
$$= 232.833$$

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$$\overline{H}(G_1) = 2\mathfrak{S}_r\mathfrak{J}(f(r,s))(1,1) = \left(\frac{20}{4}rs^3 + \frac{66}{5}r^2s^3 + \frac{128}{6}r^3s^3\right)(1,1) = 39.533$$

$$\overline{ISI}(G_1) = \mathfrak{S}_r\mathfrak{J}\mathfrak{D}_r\mathfrak{D}_s(f(r,s))(1,1) = \left(\frac{30}{4}r^4 + \frac{198}{5}s^5 + \frac{576}{6}r^6\right)(1) = 143.1$$

$$\bar{A}(G_1) = \mathfrak{S}_r^3\mathfrak{Q}_{-2}\mathfrak{J}\mathfrak{D}_r^3\mathfrak{D}_s^3(f(r,s))(1,1) = \left(\frac{270}{8}r^2 + \frac{7128}{27}r^3 + \frac{46656}{64}r^4\right)(1) = 1026.75$$

Likewise, by Table 2, we have the following;

$$NM_1(G_1) = (\mathfrak{D}_r+\mathfrak{D}_s)\big(NM(G_1)\big)|(r=s=1) = (9r^3s^6 + 12r^6s^6 + 15r^6s^9 + 56r^6s^8 + 85r^8s^9 + 14r^6s^8 + 13r^4s^9 + 16r^7s^9 + 24r^4s^8 + 22r^3s^8 + 7r^3s^4 + 32r^8s^8 + 13r^6s^7 + 14r^7s^7 + 18r^9s^9)|(r = s = 1) = 340$$

$$NM_2(G_1) = \mathfrak{D}_r\mathfrak{D}_s\big(NM(G_1)\big)|(r=s=1) = (18r^3s^6 + 36r^6s^6 + 54r^6s^9 + 192r^6s^8 + 360r^8s^9 + 48r^6s^8 + 36r^4s^9 + 63r^7s^9 + 64r^4s^8 + 42r^3s^8 + 12r^3s^4 + 128r^8s^8 + 42r^6s^7 + 49r^7s^7 + 81r^9s^9)|(r = s = 1) = 1225$$

$$NF(G_1) = (\mathfrak{D}_r^2+\mathfrak{D}_s^2)\big(NM(G_1)\big)|(r=s=1) = (45r^3s^6 + 72r^6s^6 + 117r^6s^9 + 400r^6s^8 + 725r^8s^9 + 100r^6s^8 + 97r^4s^9 + 130r^7s^9 + 160r^4s^8 + 146r^3s^8 + 25r^3s^4 + 256r^8s^8 + 85r^6s^7 + 98r^7s^7 + 16r^9s^9)|(r = s = 1) = 2618$$

$$NM^*(G_1) = \mathfrak{S}_r\mathfrak{S}_s\big(NM(G_1)\big)|(r=s=1) = \left(\frac{1}{18}r^3s^6 + \frac{1}{36}r^6s^6 + \frac{1}{54}r^6s^9 + \frac{4}{48}r^6s^8 + \frac{5}{72}r^8s^9 + \frac{1}{48}r^6s^8 + \frac{1}{36}r^4s^9 + \frac{1}{63}r^7s^9 + \frac{2}{32}r^4s^8 + \frac{2}{24}r^3s^8 + \frac{1}{12}r^3s^4 + \frac{2}{64}r^8s^8 + \frac{1}{49}r^6s^7 + \frac{1}{42}r^7s^7 + \frac{1}{81}r^9s^9\right)|(r = s = 1) = 0.6361$$

$$NRZ(G_1) = (\mathfrak{D}_r\mathfrak{D}_s)(\mathfrak{D}_r+\mathfrak{D}_s)\big(NM(G_1)\big)|(r=s=1) = (162r^3s^6 + 432r^6s^6 + 810r^6s^9 + 2688r^6s^8 + 6180r^8s^9 + 672r^6s^8 + 468r^4s^9 + 1008r^7s^9 + 768r^4s^8 + 528r^3s^8 + 84r^3s^4 + 2048r^8s^8 + 546r^6s^7 + 686r^7s^7 + 1458r^9s^9)|(r = s = 1) = 18478$$

$$NSD(G_1) = (\mathfrak{D}_r\mathfrak{S}_s+\mathfrak{S}_r\mathfrak{D}_s)\big(NM(G_1)\big)|(r=s=1) = \left(\left(\frac{3}{6}+\frac{6}{3}\right)r^3s^6 + \left(\frac{6}{6}+\frac{6}{6}\right)r^6s^6 + \left(\frac{6}{9}+\frac{9}{6}\right)r^6s^9 + \left(\frac{24}{8}+\frac{32}{6}\right)r^6s^8 + \left(\frac{40}{9}+\frac{45}{8}\right)r^8s^9 + \left(\frac{6}{8}+\frac{8}{6}\right)r^6s^8 + \left(\frac{4}{9}+\frac{9}{4}\right)r^4s^9 + \left(\frac{7}{9}+\frac{9}{7}\right)r^7s^9 + \left(\frac{8}{8}+\frac{16}{4}\right)r^4s^8 + \left(\frac{6}{8}+\frac{16}{3}\right)r^3s^8 + \left(\frac{3}{4}+\frac{4}{3}\right)r^3s^4 + \left(\frac{8}{8}+\frac{8}{8}\right)r^8s^8 + \left(\frac{6}{7}+\frac{7}{6}\right)r^6s^7 + \left(\frac{7}{7}+\frac{7}{7}\right)r^7s^7 + \left(\frac{9}{9}+\frac{9}{9}\right)r^9s^9\right)|(r = s = 1) = 57.546$$

$$NH(G_1) = 2\mathfrak{S}_r\mathfrak{J}\big(NM(G_1)\big)|(r=1) = \left(\frac{2}{9}r^9 + \frac{2}{12}r^{12} + \frac{2}{15}r^{15} + \frac{8}{14}r^{14} + \frac{10}{17}r^{17} + \frac{2}{14}r^{14} + \frac{2}{13}r^{13} + \frac{2}{16}r^{16} + \frac{4}{12}r^{12} + \frac{4}{11}r^{11} + \frac{2}{7}r^7 + \frac{4}{16}r^{16} + \frac{2}{13}r^{13} + \frac{2}{14}r^{14} + \frac{2}{18}r^{18}\right)|(r = 1) = 3.744$$

$$S(G_1) = \mathfrak{S}_r^3\mathfrak{Q}_{-2}\mathfrak{J}\,\mathfrak{D}_r^3\mathfrak{D}_r^3\big(NM(G_1)\big)|(r=1) = \left(\frac{5832}{343}r^7 + \frac{46656}{1000}r^{10} + \frac{157464}{2197}r^{13} + \frac{442368}{1728}r^{12} + \frac{1866240}{3375}r^{15} + \frac{110592}{1728}r^{12} + \frac{46656}{1331}r^{11} + \frac{250047}{2744}r^{14} + \frac{65536}{1000}r^{10} + \frac{27648}{729}r^9 + \frac{1728}{125}r^5 + \frac{524288}{2744}r^{14} + \frac{74088}{1331}r^{11} + \frac{117649}{1728}r^{12} + \frac{531441}{4096}r^{16}\right)|(r=1) = 1696.3162.$$

5. Cicada Species and its polynomials

Theorem 2. If G_2 is a wing graph of cicada insects, then $\overline{M}(G_2;r,s) = 52r^2s^3 + 341r^3s^3 + 24r^3s^4$ and $NM(G_2;r,s) = 3r^6s^8 + 3r^{10}s^{12} + r^9s^{12} + 6r^9s^{10} + 2r^8s^8 + 2r^8s^9 + 27r^9s^9 + r^6s^9$.

Proof. The wing graph G_2 of Cicada insect consist 31 vertices and 45 edges. There are 1 vertex of degree 4, 2 vertices of degree 2 and 28 vertices of degree 3, that is,

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$n_2 = |s_2| = 2, n_3 = |s_3| = 3 \text{ and } n_4 = |s_4| = 1.$

Similarly, $E(G_2)$ can also be split into three classes based on the degree of vertices:

$$E_{23} = \{rs \in E(G_2) | \zeta(r) = 2, \zeta(s) = 3\}$$
$$E_{33} = \{rs \in E(G_2) | \zeta(r) = \zeta(s) = 3\}$$
$$E_{34} = \{rs \in E(G_2) | \zeta(r) = 3, \zeta(s) = 4\}$$

Such that $m_{23} = |E_{23}| = 4, m_{33} = |E_{33}| = 37$ and $m_{34} = |E_{34}| = 4$ Using Lemma 1, we have

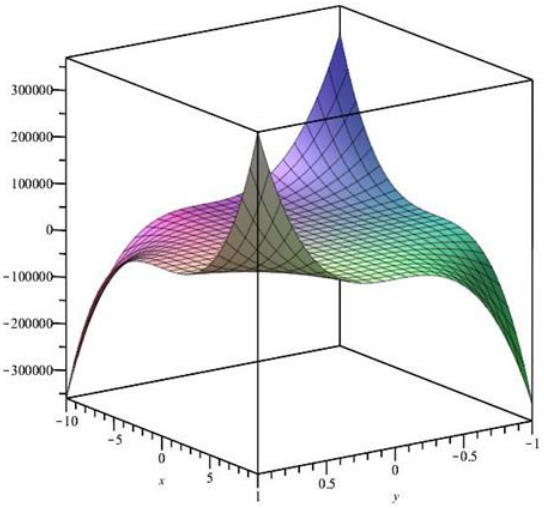
$$\overline{m_{23}} = n_2 n_3 - m_{23} = 2(28) - 4 = 52$$
$$\overline{m_{33}} = \frac{n_3(n_3 - 1)}{2} - m_{33} = \frac{28(27)}{2} - 37 = 341$$
$$\overline{m_{34}} = n_3 n_4 - m_{34} = 28(1) - 4 = 24$$

By definition of $\overline{\mathrm{M}}$ - Polynomial,

$$\overline{M}(G_2; r, s) = \sum_{2 \le 3} \overline{m_{23}}\, r^2 s^3 + \sum_{3 \le 3} \overline{m_{33}}\, r^3 s^3 + \sum_{3 \le 4} \overline{m_{34}}\, r^3 s^4$$
$$= 52r^2s^3 + 341r^3s^3 + 24r^3s^4.$$

From Figure 2, we have $\chi_{6,8} = 3, \chi_{12,10} = 3, \chi_{12,9} = 1, \chi_{9,10} = 6, \chi_{8,8} = 2, \chi_{8,9} = 2,$ $\chi_{9,9} = 27$ and $\chi_{6,9} = 1$. The NM-polynomial of G_2 is

$$NM(G_2; r, s) = \sum_{i \le j} \chi_{a,b}(G_2) r^a s^b$$
$$= \chi_{6,8} r^6 s^8 + \chi_{10,12} r^{10} s^{12} + \chi_{9,12} r^9 s^{12} + \chi_{9,10} r^9 s^{10} + \chi_{8,8} r^8 s^8 + \chi_{8,9} r^8 s^9 + \chi_{9,9} r^9 s^9 + \chi_{6,9} r^6 s^9$$
$$= 3r^6s^8 + 3r^{10}s^{12} + r^9s^{12} + 6r^9s^{10} + 2r^8s^8 + 2r^8s^9 + 27r^9s^9 + r^6s^9.$$



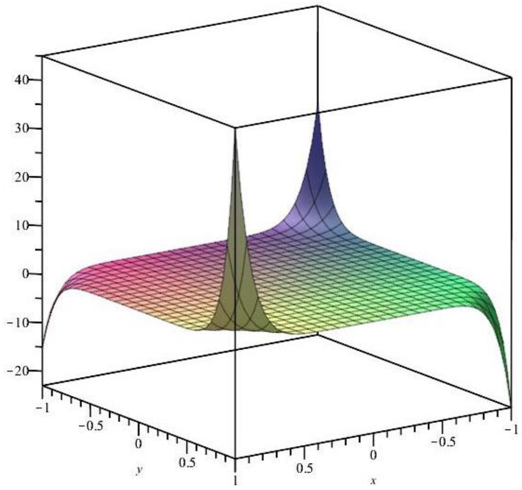


Figure 6. 3D-plots for $\overline{\mathrm{M}}$ -polynomial and NM-polynomial of wing graph of cicada

The 3D- Plots of $\overline{\mathrm{M}}$ -polynomial and NM-polynomial of wing graph of cicada species showed different patterns in Figure 6. Now we present some degree-based and neighbourhood degree- based TDs of wing graph for Cicada insects using $\overline{\mathrm{M}}$ -polynomial and NM-polynomial. From Table 1, we obtain the following;

$$\bar{M}_1(G_2) = (\mathfrak{D}_r + \mathfrak{D}_s)(f(r,s))(1,1) = 2474$$
$$\bar{M}_2(G_2) = \mathfrak{D}_r \mathfrak{D}_s(f(r,s))(1,1) = 3669$$
$$m\bar{M}_2(G_2) = \mathfrak{S}_r \mathfrak{S}_s\big(f(r,s)\big)(1,1) = 48.556$$
$$\overline{RZ}(G_2) = (\mathfrak{D}_r \mathfrak{D}_s)(\mathfrak{D}_r + \mathfrak{D}_s)\big(f(r,s)\big)(1,1) = 21990$$
$$\bar{F}(G_2) = (\mathfrak{D}_r^2 + \mathfrak{D}_s^2)(f(r,s))(1,1) = 7414$$
$$\overline{SDD}(G_2) = (\mathfrak{D}_r \mathfrak{S}_s + \mathfrak{S}_r \mathfrak{D}_s)\big(f(r,s)\big)(1,1) = 862$$
$$\bar{H}(G_2) = 2\mathfrak{S}_r \mathfrak{J}(f(r,s))(1) = 141.323$$
$$\overline{ISI}(G_2) = \mathfrak{S}_r \mathfrak{J}\, \mathfrak{D}_r \mathfrak{D}_s\big(f(r,s)\big)(1) = 615.042$$
$$\bar{A}(G_2) = \mathfrak{S}_r^3 \mathfrak{Q}_{-2} \mathfrak{J} D_r^3 \mathfrak{D}_s^3\big(f(r,s)\big)(1) = 4631.979$$

From Table 2, we obtain,

$$NM_1(G_2) = (\mathfrak{D}_r + \mathfrak{D}_s)\big(NM(G_2)\big)(1,1) = 810$$
$$NM_2(G_2) = (\mathfrak{D}_r \mathfrak{D}_s\big(NM(G_2)\big)(1,1) = 3665$$
$$NF(G_2) = ((\mathfrak{D}_r^2 + \mathfrak{D}_s^2)\big(NM(G_2)\big)(1,1) = 7380$$
$$NM^*(G_2) = (\mathfrak{S}_r \mathfrak{S}_s\big(NM(G_2)\big)(1,1) = 0.5743$$
$$NRZ(G_2) = ((\mathfrak{D}_r \mathfrak{D}_s)(\mathfrak{D}_r + \mathfrak{D}_s)\big(NM(G_2)\big)(1,1) = 67136$$
$$NSD(G_2) = ((\mathfrak{D}_r \mathfrak{S}_s + \mathfrak{D}_s \mathfrak{S}_r)\big(NM(G_2)\big)(1,1) = 89.944$$
$$NH(G_2) = (2\mathfrak{S}_r \mathfrak{J}\big(NM(G_2)\big)(1,1) = 5.0467$$
$$S(G_2) = (\mathfrak{S}_r^3\, \mathfrak{Q}_2 \mathfrak{J}\, \mathfrak{D}_r^3 \mathfrak{D}_s^3\big(NM(G_2)\big)(1,1) = 5901.024$$

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6. Topological descriptors for Apis species

Now we present two graph polynomials for Apis species. Using these polynomials we obtain the several TDs values.

Theorem 3. Let G_3 be the wing graph of Honey Bee (Apis Species). Then $\overline{M}(G_3;r,s) = 70rs^3 + 2rs^4 + 48r^2s^3 + 131r^3s^3 + 16r^3s^4$ and $NM(G_3;r,s) = 2r^4s^8 + 5r^6s^8 + 2r^3s^7 + 9r^8s^9 + 12r^9s^9 + 3r^7s^9 + r^7s^8$.

Proof. The wing graph G_3 of Honey Bee consist 26 vertices and 34 edges. Also, $V(G_3)$ can be split into four classes based on the degree they have $n_1 = |s_1| = 4, n_2 = |s_2| = 3, n_3 = |s_3| = 18$ and $n_4 = |s_4| = 1$. Similarly, $E(G_3)$ can also be split into five categories based on the degree of vertices as $m_{13} = |E_{13}| = 2, m_{14} = |E_{14}| = 2, m_{23} = |E_{23}| = 6, m_{33} = |E_{33}| = 22$, and $m_{34} = |E_{34}| = 2$.

Using Lemma 1, we have

$$\overline{m_{13}} = n_1n_3 - m_{13} = 4(18) - 2 = 70$$
$$\overline{m_{14}} = n_1n_4 - m_{14} = 4(1) - 2 = 2$$
$$\overline{m_{23}} = n_2n_3 - m_{23} = 3(18) - 6 = 48$$
$$\overline{m_{33}} = \frac{n_3(n_3-1)}{2} - m_{33} = \frac{18(17)}{2} - 22 = 131$$
$$\overline{m_{34}} = n_3n_4 - m_{34} = 18(1) - 2 = 16$$

By definition of $\overline{M}$ - Polynomial, we obtain

$$\overline{M}(G_3;r,s) = \sum_{1\le 3}\overline{m_{13}}\,rs^3 + \sum_{1\le 4}\overline{m_{14}}\,rs^4 + \sum_{2\le 3}\overline{m_{23}}\,r^2s^3 + \sum_{3\le 3}\overline{m_{33}}\,r^3s^3 + \sum_{3\le 4}\overline{m_{34}}\,r^3s^4$$
$$= 70rs^3 + 2rs^4 + 48r^2s^3 + 131r^3s^3 + 16r^3s^4.$$

From Figure 3, we have $\chi_{4,8} = 2, \chi_{6,8} = 5, \chi_{3,7} = 2, \chi_{8,9} = 9, \chi_{9,9} = 12, \chi_{7,9} = 3$, and $\chi_{7,8} = 1$. The NM-polynomial of G_2 is

$$NM(G_3;r,s) = \sum_{i\le j}\chi_{a,b}(G_3)r^as^b$$
$$= \chi_{4,8}r^4s^8 + \chi_{6,8}r^6s^8 + \chi_{3,7}r^3s^7 + \chi_{8,9}r^8s^9 + \chi_{9,9}r^9s^9 + \chi_{7,9}r^7s^9 + \chi_{7,8}r^7s^8.$$
$$= 2r^4s^8 + 5r^6s^8 + 2r^3s^7 + 9r^8s^9 + 12r^9s^9 + 3r^7s^9 + r^7s^8.$$

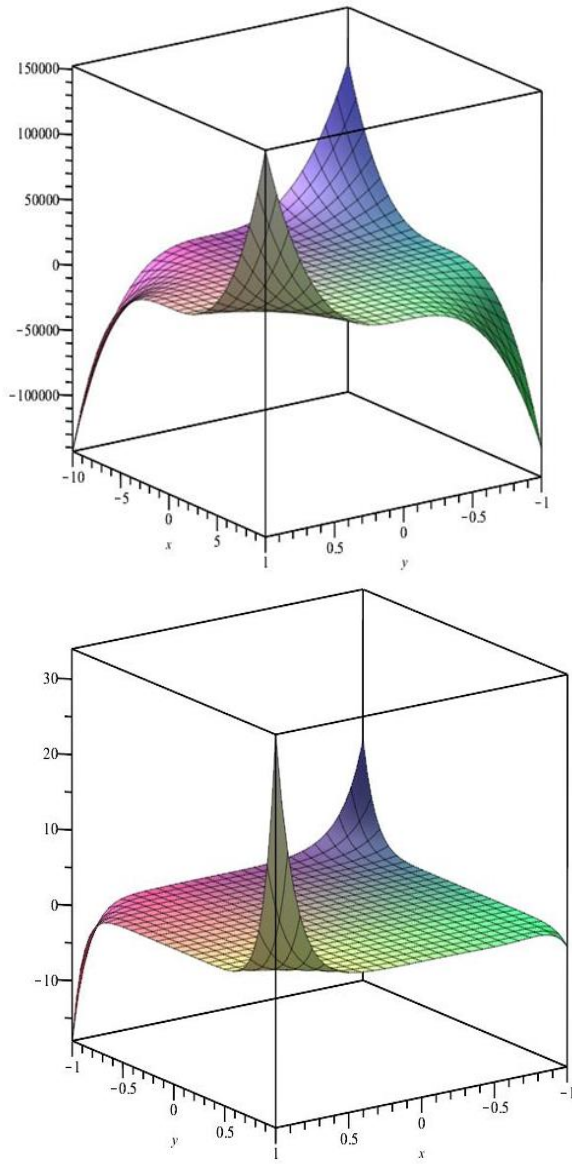

Figure 7. 3D-plots for $\overline{\text{M}}$ -polynomial and NM-polynomial of wing graph of Honey Bee

The 3D- Plots of $\overline{\text{M}}$ -polynomial and NM-polynomial of wing graph of Honey Bee species showed different patterns in Figure 7. Now we present some degree-based and neighbourhood degree- based TDs of wing graph for Honey Bee using $\overline{\text{M}}$ -polynomial and NM-polynomial. From Table 1, we obtain the following;

$$\overline{M}_1(G_3) = (\mathfrak{D}_r + \mathfrak{D}_s)(f(r,s))(1,1) = 1428$$
$$\overline{M}_2(G_3) = \mathfrak{D}_r\mathfrak{D}_s(f(r,s))(1,1) = 1877$$
$$m\overline{M}_2(G_3) = \mathfrak{S}_r\mathfrak{S}_s(f(r,s))(1,1) = 47.722$$
$$\overline{RZ}(G_3) = (\mathfrak{D}_r\mathfrak{D}_s)(\mathfrak{D}_r + \mathfrak{D}_s)(f(r,s))(1,1) = 10738$$
$$\overline{F}(G_3) = (\mathfrak{D}_r^2 + \mathfrak{D}_s^2)(f(r,s))(1,1) = 4116$$
$$\overline{SDD}(G_3) = (\mathfrak{D}_r\mathfrak{S}_s + \mathfrak{S}_r\mathfrak{D}_s)(f(r,s))(1,1) = 641.1667$$

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$$\overline{H}(G_3) = 2\mathfrak{S}_r\mathfrak{J}(f(r,s))(1,1) = 103.238$$
$$\overline{ISI}(G_3) = \mathfrak{S}_r\mathfrak{J}\mathfrak{D}_r\mathfrak{D}_s(f(r,s))(1,1) = 335.628$$
$$\bar{A}(G_3) = \mathfrak{S}_r^3\mathfrak{Q}_{-2}\mathfrak{J}\mathfrak{D}_r^3\mathfrak{D}_s^3(f(r,s))(1,1) = 2338.346$$

From Table 2, we have

$$NM_1(G_3) = (\mathfrak{D}_r + \mathfrak{D}_s)(NM(G_3))(1,1) = 546$$
$$NM_2(G_3) = \mathfrak{D}_r\mathfrak{D}_s(NM(G_3))(1,1) = 2211$$
$$NF(G_3) = (\mathfrak{D}_r^2 + \mathfrak{D}_s^2)(NM(G_3))(1,1) = 4528$$
$$NM^*(G_3) = \mathfrak{S}_r\mathfrak{S}_s(NM(G_3))(1,1) = 0.60053$$
$$NRZ(G_3) = (\mathfrak{D}_r\mathfrak{D}_s)(\mathfrak{D}_r + \mathfrak{D}_s)(NM(G_3))(1,1) = 36924$$
$$NSD(G_3) = (\mathfrak{D}_r\mathfrak{S}_s + \mathfrak{D}_s\mathfrak{S}_r)(NM(G_3))(1,1) = 71.274$$
$$NH(G_3) = 2\mathfrak{S}_r\mathfrak{J}(NM(G_3))(1) = 4.3481$$
$$S(G_3) = \mathfrak{S}_r^3\mathfrak{Q}_{-2}\mathfrak{J}\,\mathfrak{D}_r^3\mathfrak{D}_s^3(NM(G_3))(1,1) = 3327.3053$$

7. Musca species and their TDs

Theorem 4. If G_4 be the wing graph of Musca Species, Then

$$\overline{M}(G_4;r,s) = 80rs^3 + 20rs^4 + 5rs^5 + 2r^2s^2 + 34r^2s^3 + 8r^2s^4 + 2r^2s^5 + 55r^3s^3 + 29r^3s^4 + 11r^3s^5 + 2r^4s^4 + 2r^4s^5$$

and

$$NM(G_4;r,s) = 2r^3s^8 + r^3s^7 + 2r^6s^8 + 3r^9s^{10} + 4r^8s^9 + 3r^4s^{10} + r^4s^{11} + r^{11}s^{14} + r^6s^6 + r^{10}s^{14} + 2r^{11}s^{13} + r^6s^{13} + r^{13}s^{14} + r^{10}s^{11} + r^9s^{11} + r^7s^9 + 2r^8s^{10} + r^6s^{11} + 2r^{10}s^{11} + 2r^{10}s^{10} + r^7s^8 + r^8s^{14}.$$

Proof. The wing graph G_4 has 26 vertices and 33 edges. There are 7 vertices of degree1, 3 vertices of degree 2, 12 vertices of degree 3, 3 vertices of degree 4 and 1 vertex of degree5, that is $n_1 = |s_1| = 7, n_2 = |s_2| = 3, n_3 = |s_3| = 12, n_4 = |s_4| = 3$ and $n_5 = |s_5| = 1$. Similarly, the edge set of G_4can split 12 subsets such as $m_{13} = 4, m_{14} = 1, m_{15} = 2, m_{22} = 1, m_{23} = 2, m_{24} = 1, m_{25} = 1, m_{33} = 11, m_{34} = 7, m_{35} = 1, m_{44} = 1$ and $m_{45} = 1$.

Using Lemma 1, we have $\overline{m_{13}} = 80, \overline{m_{14}} = 20, \overline{m_{15}} = 5, \overline{m_{22}} = 2, \overline{m_{23}} = 34, \overline{m_{24}} = 8, \overline{m_{25}} = 2, \overline{m_{33}} = 55, \overline{m_{34}} = 29, \overline{m_{35}} = 11, \overline{m_{44}} = 2$ and $\overline{m_{45}} = 2$. Hence

$$\overline{M}(G_4;r,s) = 80rs^3 + 20rs^4 + 5rs^5 + 2r^2s^2 + 34r^2s^3 + 8r^2s^4 + 2r^2s^5 + 55r^3s^3 + 29r^3s^4 + 11r^3s^5 + 2r^4s^4 + 2r^4s^5.$$

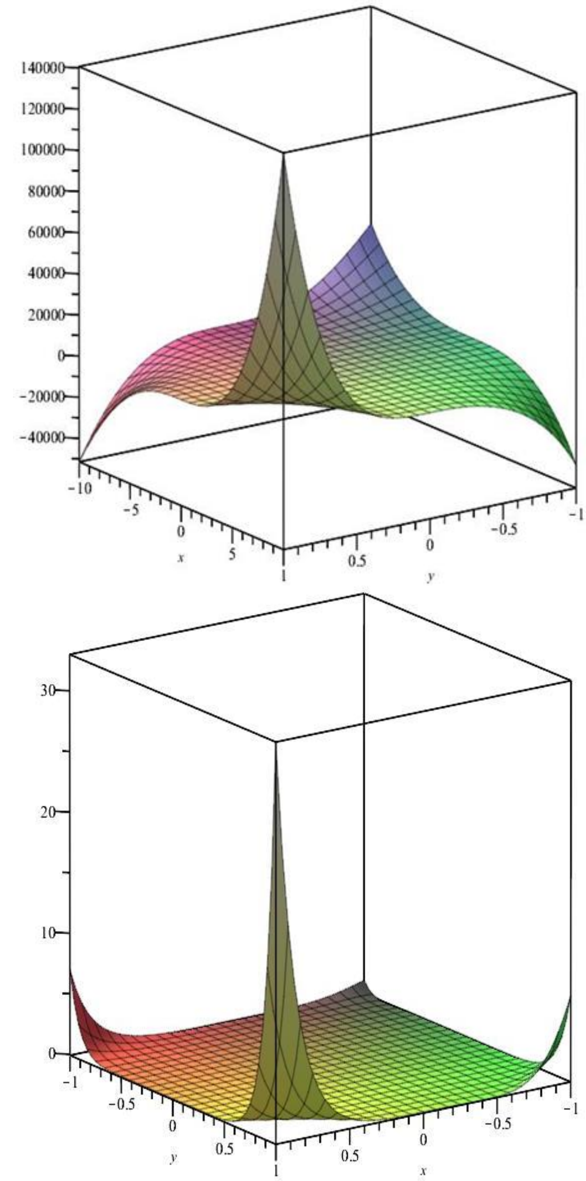

Figure 8. 3D-plots for $\overline{\mathrm{M}}$ -polynomial and NM-polynomial of wing graph of Musca

From Figure 4, we have $\chi_{3,8} = 2, \chi_{3,7} = 1, \chi_{6,8} = 2, \chi_{9,10} = 3, \chi_{8,9} = 4, \chi_{4,10} = 3, \chi_{4,11} = 1, \chi_{11,14} = 1, \chi_{6,6} = 1, \chi_{10,4} = 1, \chi_{11,13} = 2, \chi_{6,13} = 1, \chi_{13,14} = 1, \chi_{10,11} = 1, \chi_{9,11} = 1, \chi_{7,9} = 1, \chi_{8,10} = 2, \chi_{6,11} = 1, \chi_{10,10} = 2, \chi_{7,8} = 1$, and $\chi_{8,14} = 1$. Hence

$$NM(G_4;r,s) = \sum_{i \le j} \chi_{a,b}(G_4) r^a s^b$$

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$$= 2r^3s^8 + r^3s^7 + 2r^6s^8 + 3r^9s^{10} + 4r^8s^9 + 3r^4s^{10} + r^4s^{11} + r^{11}s^{14} + r^6s^6 + r^{10}s^{14} + 2r^{11}s^{13} + r^6s^{13} + r^{13}s^{14} + r^{10}s^{11} + r^9s^{11} + r^7s^9 + 2r^8s^{10} + r^6s^{11} + 2r^{10}s^{11} + 2r^{10}s^{10} + r^7s^8 + r^8s^{14}.$$

The 3D- Plots of $\overline{M}$ -polynomial and NM-polynomial of wing graph of Musca species showed different patterns in Figure 8. Now we present some degree-based and neighbourhood degree- based TDs of wing graph for Musca using $\overline{M}$ -polynomial and NM-polynomial. From Table 1, we obtain the following;

$$\bar{M}_1(G_4) = (\mathfrak{D}_r + \mathfrak{D}_s)(f(r,s))(1,1) = 1345$$
$$\bar{M}_2(G_4) = \mathfrak{D}_r\mathfrak{D}_s(f(r,s))(1,1) = 1721$$
$$m\bar{M}_2(G_4) = \mathfrak{S}_r\mathfrak{S}_s(f(r,s))(1,1) = 49.519$$
$$\overline{RZ}(G_4) = (\mathfrak{D}_r\mathfrak{D}_s)(\mathfrak{D}_r + \mathfrak{D}_s)(f(r,s))(1,1) = 15896$$
$$\bar{F}(G_4) = (\mathfrak{D}_r^2 + \mathfrak{D}_s^2)(f(r,s))(1,1) = 4181$$
$$\overline{SDD}(G_4) = (\mathfrak{D}_r\mathfrak{S}_s + \mathfrak{S}_r\mathfrak{D}_s)(f(r,s))(1,1) = 684.583$$
$$\bar{H}(G_4) = 2\mathfrak{S}_r\mathfrak{J}(f(r,s))(1,1) = 97.8183$$
$$\overline{ISI}(G_4) = \mathfrak{S}_r\mathfrak{J}\mathfrak{D}_r\mathfrak{D}_s(f(r,s))(1,1) = 297.774$$
$$\bar{A}(G_4) = \mathfrak{S}_r^3\mathfrak{Q}_2\mathfrak{J}\mathfrak{D}_r^3\mathfrak{D}_s^3(f(r,s))(1,1) = 1979.0015$$

Using Table 2, we get

$$NM_1(G_4) = (\mathfrak{D}_r + \mathfrak{D}_s)(NM(G_4))(1,1) = 584$$
$$NM_2(G_4) = \mathfrak{D}_r\mathfrak{D}_s(NM(G_4))(1,1) = 2631$$
$$NF(G_4) = (\mathfrak{D}_r^2 + \mathfrak{D}_s^2)(NM(G_4))(1,1) = 5650$$
$$NM^*(G_4) = \mathfrak{S}_r\mathfrak{S}_s(NM(G_4))(1,1) = 0.5549$$
$$NRZ(G_4) = (\mathfrak{D}_r\mathfrak{D}_s)(\mathfrak{D}_r + \mathfrak{D}_s)(NM(G_4))(1,1) = 51990$$
$$NSD(G_4) = (\mathfrak{D}_r\mathfrak{S}_s + \mathfrak{S}_r\mathfrak{D}_s)(NM(G_4))(1,1) = 74.511$$
$$NH(G_4) = 2\mathfrak{S}_r\mathfrak{J}(NM(G_4))(1,1) = 3.9579$$
$$S(G_4) = \mathfrak{S}_r^3\mathfrak{Q}_2\mathfrak{J}\mathfrak{D}_r^3\mathfrak{D}_s^3(NM(G_4))(1,1) = 4242.837$$

Observation and conclusion:

All sexually reproducing animals have genetic variety, which results in variances and differentiation at all levels of body structure, morphology, behaviour, and ecology, among other things. It is believed that by calculating topological descriptors for wings, insect orders can be identified and different insects can be distinguished based on TDs. Venation pattern in the wings of all insects showed different morphometric parameters like venation pattern, distribution, and distance between vertices. For a very long period, distinct insect species were identified using morphological characteristics and the pattern of their wings [1]. In the current work, topological descriptors were used for the first time to distinguish between insects from different species and to identify insect orders.

For the diverse species of Drosophila, Cicada, Apis, and Musca, the estimated values of the first Zagreb index were 589, 2474, 1428, and 1345, respectively. These values varied depending on the species. In a similar manner, the estimated values for the second Zagreb coindex, the Modified second Zagreb coindex, and the Redefined Zagreb coindex vary depending on the species of insects, demonstrating the significance and power of topological descriptors to distinguish insects. However, estimation of morphological analysis is a simple approach and does not require any expensive special equipment, making it relevant [14, 15]. Identification of insects by molecular methods is currently utilized as a confirmatory test, although the procedure is highly expensive. In earlier studies, the Cubital index of the fore wing of several bee species was estimated [11]. Since many years ago, morphometric analysis has regularly employed the cubital and discoidal indices of wing analysis.

A study demonstrated the effectiveness of wing geometry in identifying insects from various orders within the insecta class of the Arthropoda phylum. It is possible to identify various insects by estimating their topological indices. This research created a brand-new avenue for multidisciplinary research involving mathematics and entomology.

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