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Cube Difference Labeling of an Extended Grid

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KEYWORDS

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ABSTRACT:

Graph labeling is a task of assigning integers to the vertices or edges or both subject to certain conditions. In this paper we prove that the extended grid EM(1,n) admits Cube difference labeling.

1. Introduction

Several methods of labeling in graphs have evolved and serve as beneficial models with wide range of applications in diverse fields such as technology etc.. Prominent among the types of labeling is cube difference labelling [1]. A useful survey on graph labeling by J.A.Gallian (2019) can be found in [5]. In this paper we deal only finite, simple, connected and undirected graphs.For number theory concepts refer [3] some of the basic definitions are given below

Definition [1] **1.1.** Let $G = \big(V(G), E(G)\big)$ be a graph. G is said to be a cube difference labeling if there exists a bijection $f: V(G) \to \{0,1,2,\ldots\ldots,p-1\}$ such that the induced function
$f^*: E(G) \to N$ is given by
$f^*(uv) = |[f(u)]^3 - [f(v)]^3|$ for every $uv \in E(G)$ are all distinct. Any graph which admits cube difference labeling is said to be cube difference labeling graph.

Definition [6], [7] **1.2.** A *two-dimensional grid* (also called a Mesh) $M(r,s)$ is a graph whose vertex set is the set of ordered pairs on nonnegative integers, $\{(i,j): 0 \leq i < r, 0 \leq j < s\}$, in which there is an edge between vertices (i,j) and (k,l) if either $|i-k| = 1$ and $j = l$ or $i = k$ and $|j-l| = 1$. For any i, $0 \leq i < r$, the subset of vertices $\{(i,j): 0 \leq j < s\}$ will be called the i^{th} row of the grid. For any j, $0 \leq j < s$, j^{th} the column is similarly defined as the set $\{(i,j): 0 \leq i < r\}$.

Definition [6], [7] **1.3.** The *extended grid* $EM(r,s)$ is a graph whose vertex set is the set of pairs on nonnegative integers, $\{(i,j): 0 \leq i < r, 0 \leq j < s\}$, in which there is an edge between vertices(i,j) and (k,l) if and only if $|i-k| \leq 1$ and $|j-l| \leq 1$. Thus, the extended grid is obtained from a *two-dimensional grid* by adding diagonal edges to the nodes. The graph $EM(m,n)$ consists of m rows of nK_4 graphs and n columns of mK_4 graphs.

2. Results

Theorem 2.1.

The extended grid $EM(1,n)$ admits cube difference labeling, for $n \geq 2$.

Proof:

Let $EM(1,n)$ be an extended grid with 1 row and n columns or nK_4 graphs.

We denote the extended grid $EM(1,n)$ by G having vertices
$v_0, v_1, v_2, \ldots\ldots\ldots, v_{2n+1}$, and
edges $e_1, e_2, \ldots\ldots\ldots, e_{5n+1}$.

We find that

$|V(G)| = 2(n+1)$,
$|E(G)| = 5n+1$.

Define $\mathcal{P}: V(G) \to \{0,1,2,\ldots\ldots,2n+1\}$

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by $\mathcal{P}(v_i) = i, 0 \le i < 2n+1$

$\mathcal{P}$ induces a cube difference labeling on G.

For if, $\mathcal{P}^*$ be the induced function defined

by

$\mathcal{P}^*: E(G) \to N$ such that

$\mathcal{P}^*(v_l v_m) = |\,[\mathcal{P}(v_l)]^3 - [\mathcal{P}(v_m)]^3\,|$

Let $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ Were

$E_1 = \{e_s / e_s = v_{2s-1}v_{2s+1}, 1 \le s \le n\}$

$E_2 = \{e_s / e_s = v_{2s-2}v_{2s}, 1 \le s \le n\}$

$E_3 = \{e_s / e_s = v_{2s-2}v_{2s-1}, 1 \le s \le n+1\}$

$E_4 = \{e_s / e_s = v_{2s-2}v_{2s+1}, 1 \le s \le n\}$

$E_5 = \{e_s / e_s = v_{2s-1}v_{2s}, 1 \le s \le n\}$.

To prove that $\mathcal{P}^*$ is injective in E.

***Claim* 1:** $\mathcal{P}^*$ is injective in E_1.

Let $e_1, e_2, \ldots\ldots\ldots, e_n$ be the n edges of E_1.

It is visible that

$$\mathcal{P}(v_1) < \mathcal{P}(v_3) < \mathcal{P}(v_5) < \cdots < \mathcal{P}(v_{2n-1}) < \mathcal{P}(v_{2n+1})$$

Then
$[\mathcal{P}(v_1)]^3 < [\mathcal{P}(v_3)]^3 < [\mathcal{P}(v_5)]^3 < \ldots\ldots < [\mathcal{P}(v_{2n-1})]^3 < [\mathcal{P}(v_{2n+1})]^3$

So

$|[\mathcal{P}(v_1)]^3 - [\mathcal{P}(v_3)]^3| < |[\mathcal{P}(v_3)]^3 - [\mathcal{P}(v_5)]^3| < \cdots < |[\mathcal{P}(v_{2n-1})]^3 - [\mathcal{P}(v_{2n+1})]^3|$
Hence
$\mathcal{P}^*(v_1v_3) < \mathcal{P}^*(v_3v_5) < \cdots < \mathcal{P}^*(v_{2n-1}v_{2n+1})$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2) < \cdots < \mathcal{P}^*(e_n)$
Thus $\mathcal{P}^*$ is injective in E_1.
Hence all the edge labelings in E_1 are distinct.
***Claim* 2:** $\mathcal{P}^*$ is in injective in E_2.
Let $e_1, e_2, \ldots\ldots\ldots, e_n$ be the n edges of E_2.
It is clear that

$\mathcal{P}(v_0) < \mathcal{P}(v_2) < \mathcal{P}(v_4) < \cdots < \mathcal{P}(v_{2n-2}) < \mathcal{P}(v_{2n})$

Then

$[\mathcal{P}(v_0)]^3 < [\mathcal{P}(v_2)]^3 < [\mathcal{P}(v_4)]^3 < \ldots\ldots < [\mathcal{P}(v_{2n-2})]^3 < [\mathcal{P}(v_{2n})]^3$
So
$|[\mathcal{P}(v_0)]^3 - [\mathcal{P}(v_2)]^3| < |[\mathcal{P}(v_2)]^3 - [\mathcal{P}(v_4)]^3| < \cdots <$

$$|[\mathcal{P}(v_{2n-2})]^3 - [\mathcal{P}(v_{2n})]^3|$$
Hence
$\mathcal{P}^*(v_0v_2) < \mathcal{P}^*(v_2v_4) < \cdots < \mathcal{P}^*(v_{2n-2}v_{2n})$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2) < \cdots < \mathcal{P}^*(e_n)$
Thus $\mathcal{P}^*$ is injective in E_2.
Hence all the edge labelings in E_2 are distinct.

***Claim* 3:** $\mathcal{P}^*$ is in injective in E_3.

Let $e_1, e_2, \ldots\ldots\ldots, e_{n+1}$ be the $n+1$ edges of E_3.
It is clear that
$\mathcal{P}(v_0) < \mathcal{P}(v_1) < \mathcal{P}(v_2) < \cdots < \mathcal{P}(v_{2n}) < \mathcal{P}(v_{2n+1})$
Then
$[\mathcal{P}(v_0)]^3 < [\mathcal{P}(v_1)]^3 < [\mathcal{P}(v_2)]^3 < \ldots\ldots < [\mathcal{P}(v_{2n})]^3 < [\mathcal{P}(v_{2n+1})]^3$
So
$|[\mathcal{P}(v_0)]^3 - [\mathcal{P}(v_1)]^3| < |[\mathcal{P}(v_2)]^3 - [\mathcal{P}(v_3)]^3| < \cdots <$

$$|[\mathcal{P}(v_{2n})]^3 - [\mathcal{P}(v_{2n+1})]^3|$$
Hence
$\mathcal{P}^*(v_0v_1) < \mathcal{P}^*(v_2v_3) < \cdots < \mathcal{P}^*(v_{2n}v_{2n+1})$

$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2) < \cdots < \mathcal{P}^*(e_{n+1})$

Thus $\mathcal{P}^*$ is injective in E_3.
Hence all the edge labelings in E_3 are distinct.
***Claim* 4:** $\mathcal{P}^*$ is in injective in E_4.

Let us consider any two edges

$e_1 = v_2v_5, e_2 = v_6v_9$ where $e_1, e_2 \in E_4$
It is visible that
$\mathcal{P}(v_2) < \mathcal{P}(v_5) < \mathcal{P}(v_6) < \mathcal{P}(v_9)$
$\Rightarrow [\mathcal{P}(v_2)]^3 < [\mathcal{P}(v_5)]^3 < [\mathcal{P}(v_6)]^3 < [\mathcal{P}(v_9)]^3$
Hence
$|[\mathcal{P}(v_2)]^3 - [\mathcal{P}(v_5)]^3| < |[\mathcal{P}(v_6)]^3 - [\mathcal{P}(v_9)]^3|$
$\mathcal{P}^*(v_2v_5) < \mathcal{P}^*(v_6v_9)$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$
$\mathcal{P}^*(e_1) \ne \mathcal{P}^*(e_2)$
Thus $\mathcal{P}^*$ is injective in E_4.

Hence all the edge labelings in E_4 are distinct.
***Claim* 5:** $\mathcal{P}^*$ is in injective in E_5.
Let $e_1, e_2, \ldots\ldots\ldots, e_n$ be the n edges of E_5.
It is clear that
$\mathcal{P}(v_1) < \mathcal{P}(v_2) < \mathcal{P}(v_3) < \cdots < \mathcal{P}(v_{2n-1}) < \mathcal{P}(v_{2n})$
Then
$[\mathcal{P}(v_1)]^3 < [\mathcal{P}(v_2)]^3 < [\mathcal{P}(v_3)]^3 <\ldots\ldots< [\mathcal{P}(v_{2n-1})]^3 < [\mathcal{P}(v_{2n})]^3$
So
$|[\mathcal{P}(v_1)]^3 - [\mathcal{P}(v_2)]^3| < |[\mathcal{P}(v_3)]^3 - [\mathcal{P}(v_4)]^3| < \cdots < |[\mathcal{P}(v_{2n-1})]^3 - [\mathcal{P}(v_{2n})]^3|$
Hence
$\mathcal{P}^*(v_1v_2) < \mathcal{P}^*(v_3v_4) < \cdots < \mathcal{P}^*(v_{2n-1}v_{2n})$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2) < \cdots < \mathcal{P}^*(e_n)$
Thus $\mathcal{P}^*$ is injective in E_5.
Hence all the edge labelings in E_5 are distinct.
***Claim* 6:** $\mathcal{P}^*$ is in injective in E_1 and E_2.
Let us consider any two edges
$e_1 = v_5v_7, e_2 = v_8v_{10}$ where $e_1 \in E_1, e_2 \in E_2$
It is visible that
$\mathcal{P}(v_5) < \mathcal{P}(v_7) < \mathcal{P}(v_8) < \mathcal{P}(v_{10})$

$\Rightarrow [\mathcal{P}(v_5)]^3 < [\mathcal{P}(v_7)]^3 < \quad [\mathcal{P}(v_8)]^3[\mathcal{P}(v_{10})]^3$

Hence
$|[\mathcal{P}(v_5)]^3 - [\mathcal{P}(v_7)]^3| < |[\mathcal{P}(v_8)]^3 - [\mathcal{P}(v_{10})]^3|$
$\mathcal{P}^*(v_5v_7) < \mathcal{P}^*(v_8v_{10})$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$
$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$
Thus $\mathcal{P}^*$ is injective in E_1 and E_2
Hence all the edge labelings in E_1 and E_2 are distinct.
***Claim* 7:** $\mathcal{P}^*$ is in injective in E_1 and E_3.
Let us consider any two edges
$e_1 = v_2v_3, e_2 = v_5v_7$ where $e_1 \in E_3, e_2 \in E_1$
It is clear that

$\mathcal{P}(v_2) < \mathcal{P}(v_3) < \mathcal{P}(v_5) < \mathcal{P}(v_7)$

$\Rightarrow [\mathcal{P}(v_2)]^3 < [\mathcal{P}(v_3)]^3 < [\mathcal{P}(v_5)]^3 < [\mathcal{P}(v_7)]^3$

Hence

$|[\mathcal{P}(v_2)]^3 - [\mathcal{P}(v_3)]^3| < |[\mathcal{P}(v_5)]^3 - [\mathcal{P}(v_7)]^3|$

$\mathcal{P}^*(v_2v_3) < \mathcal{P}^*(v_5v_7)$

$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$
$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$
Thus $\mathcal{P}^*$ is injective in E_1 and E_3
Hence all the edge labelings in E_1 and E_3 are distinct.
***Claim* 8:** $\mathcal{P}^*$ is in injective in E_1 and E_4.
Let us consider any two edges
$e_1 = v_7v_9, e_2 = v_2v_5$ where $e_1 \in E_1, e_2 \in E_4$
It is obvious that
$\mathcal{P}(v_2) < \mathcal{P}(v_5) < \mathcal{P}(v_7) < \mathcal{P}(v_9)$
$\Rightarrow [\mathcal{P}(v_2)]^3 < [\mathcal{P}(v_5)]^3 < [\mathcal{P}(v_7)]^3 < [\mathcal{P}(v_9)]^3$
Hence
$|[\mathcal{P}(v_2)]^3 - [\mathcal{P}(v_5)]^3| < |[\mathcal{P}(v_7)]^3 - [\mathcal{P}(v_9)]^3|$
$\mathcal{P}^*(v_2v_5) < \mathcal{P}^*(v_7v_9)$
$\mathcal{P}^*(e_2) < \mathcal{P}^*(e_1)$
$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$
Thus $\mathcal{P}^*$ is injective in E_1 and E_4
Hence all the edge labelings in E_1 and E_4 are distinct.
***Claim* 9:** $\mathcal{P}^*$ is in injective in E_1 and E_5.
Let us consider any two edges
$e_1 = v_3v_4, e_2 = v_9v_{11}$ where $e_1 \in E_5, e_2 \in E_1$
It is visible that
$\mathcal{P}(v_3) < \mathcal{P}(v_4) < \mathcal{P}(v_9) < \mathcal{P}(v_{11})$
$\Rightarrow [\mathcal{P}(v_3)]^3 < [\mathcal{P}(v_4)]^3 < [\mathcal{P}(v_9)]^3 < [\mathcal{P}(v_{11})]^3$
Hence $|[\mathcal{P}(v_3)]^3 - [\mathcal{P}(v_4)]^3| < |[\mathcal{P}(v_9)]^3 - [\mathcal{P}(v_{11})]^3|$

$\mathcal{P}^*(v_3v_4) < \mathcal{P}^*(v_9v_{11})$

$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$
$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$
Thus $\mathcal{P}^*$ is injective in E_1 and E_5
Hence all the edge labelings in E_1 and E_5 are distinct.
***Claim* 10:** $\mathcal{P}^*$ is in injective in E_2 and E_3.
Let us consider any two edges
$e_1 = v_4v_5, e_2 = v_8v_{10}$ where $e_1 \in E_3, e_2 \in E_2$
It is noticeable that
$\mathcal{P}(v_5) < \mathcal{P}(v_5) < \mathcal{P}(v_8) < \mathcal{P}(v_{10})$
$\Rightarrow [\mathcal{P}(v_4)]^3 < [\mathcal{P}(v_5)]^3 < [\mathcal{P}(v_8)]^3 < [\mathcal{P}(v_{10})]^3$
Hence
$|[\mathcal{P}(v_4)]^3 - [\mathcal{P}(v_5)]^3| < |[\mathcal{P}(v_8)]^3 - [\mathcal{P}(v_{10})]^3|$
$\mathcal{P}^*(v_4v_5) < \mathcal{P}^*(v_8v_{10})$

$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$

$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$

Thus $\mathcal{P}^*$ is injective in E_2 and E_3
Hence all the edge labelings in E_2 and E_3 are distinct.
***Claim* 11:** $\mathcal{P}^*$ is in injective in E_2 and E_4.
Let us consider any two edges
$e_1 = v_2v_4, e_2 = v_8v_{11}$ where $e_1 \in E_2, e_2 \in E_4$

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It is evident that
$\mathcal{P}(v_2) < \mathcal{P}(v_4) < \mathcal{P}(v_8) < \mathcal{P}(v_{11})$
$\Longrightarrow [\mathcal{P}(v_2)]^3 < [\mathcal{P}(v_4)]^3 < [\mathcal{P}(v_8)]^3 < [\mathcal{P}(v_{11})]^3$
Hence
$|[\mathcal{P}(v_2)]^3 - [\mathcal{P}(v_4)]^3| < |[\mathcal{P}(v_8)]^3 - [\mathcal{P}(v_{11})]^3|$
$\mathcal{P}^*(v_2v_4) < \mathcal{P}^*(v_8v_{11})$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$
$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$

Thus $\mathcal{P}^*$ is injective in E_2 and E_4
Hence all the edge labelings in E_2 and E_4 are distinct.
***Claim* 12:** $\mathcal{P}^*$ is in injective in E_2 and E_5.
Let us consider any two edges
$e_1 = v_2v_4, e_2 = v_9v_{10}$ where $e_1 \in E_2, e_2 \in E_5$
It is easily seen that
$\mathcal{P}(v_2) < \mathcal{P}(v_4) < \mathcal{P}(v_9) < \mathcal{P}(v_{10})$
$\Longrightarrow [\mathcal{P}(v_2)]^3 < [\mathcal{P}(v_4)]^3 < [\mathcal{P}(v_9)]^3 < [\mathcal{P}(v_{10})]^3$
Hence
$|[\mathcal{P}(v_2)]^3 - [\mathcal{P}(v_4)]^3| < |[\mathcal{P}(v_9)]^3 - [\mathcal{P}(v_{10})]^3|$
$\mathcal{P}^*(v_2v_4) < \mathcal{P}^*(v_9v_{10})$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$
$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$
Thus $\mathcal{P}^*$ is injective in E_2 and E_5
Hence all the edge labelings in E_2 and E_5 are distinct.
***Claim* 13:** $\mathcal{P}^*$ is in injective in E_3 and E_4.
Let us consider any two edges
$e_1 = v_0v_1, e_2 = v_8v_{11}$ where $e_1 \in E_3, e_2 \in E_4$
It is evident that
$\mathcal{P}(v_0) < \mathcal{P}(v_1) < \mathcal{P}(v_8) < \mathcal{P}(v_{11})$
$\Longrightarrow [\mathcal{P}(v_0)]^3 < [\mathcal{P}(v_1)]^3 < [\mathcal{P}(v_8)]^3 < [\mathcal{P}(v_{11})]^3$
Hence
$|[\mathcal{P}(v_0)]^3 - [\mathcal{P}(v_1)]^3| < |[\mathcal{P}(v_8)]^3 - [\mathcal{P}(v_{11})]^3|$
$\mathcal{P}^*(v_0v_1) < \mathcal{P}^*(v_8v_{11})$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$
$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$

Thus $\mathcal{P}^*$ is injective in E_3 and E_4
Hence all the edge labelings in E_3 and E_4 are distinct.
***Claim* 14:** $\mathcal{P}^*$ is in injective in E_3 and E_5.
Let us consider any two edges
$e_1 = v_2v_3, e_2 = v_9v_{10}$ where $e_1 \in E_3$, $e_2 \in E_5$
It is obvious that
$\mathcal{P}(v_2) < \mathcal{P}(v_3) < \mathcal{P}(v_9) < \mathcal{P}(v_{10})$
$\Longrightarrow [\mathcal{P}(v_2)]^3 < [\mathcal{P}(v_3)]^3 < [\mathcal{P}(v_9)]^3 < [\mathcal{P}(v_{10})]^3$
Hence
$|[\mathcal{P}(v_2)]^3 - [\mathcal{P}(v_3)]^3| < |[\mathcal{P}(v_9)]^3 - [\mathcal{P}(v_{10})]^3|$
$\mathcal{P}^*(v_2v_3) < \mathcal{P}^*(v_9v_{10})$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$
$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$
Thus $\mathcal{P}^*$ is injective in E_3 and E_5
Hence all the edge labelings in E_3 and E_5 are distinct.
***Claim* 15:** $\mathcal{P}^*$ is injective in E_4 and E_5.
Let us consider any two edges
$e_1 = v_3v_4, e_2 = v_8v_{11}$ where $e_1 \in E_5, e_2 \in E_4$
It is noticeable that
$\mathcal{P}(v_3) < \mathcal{P}(v_4) < \mathcal{P}(v_8) < \mathcal{P}(v_{11})$
$\Longrightarrow [\mathcal{P}(v_3)]^3 < [\mathcal{P}(v_4)]^3 < [\mathcal{P}(v_8)]^3 < [\mathcal{P}(v_{11})]^3$
Hence
$|[\mathcal{P}(v_3)]^3 - [\mathcal{P}(v_4)]^3| < |[\mathcal{P}(v_8)]^3 - [\mathcal{P}(v_{11})]^3|$
$\mathcal{P}^*(v_3v_4) < \mathcal{P}^*(v_8v_{11})$
$\mathcal{P}^*(e_1) < \mathcal{P}^*(e_2)$
$\mathcal{P}^*(e_1) \neq \mathcal{P}^*(e_2)$
Thus $\mathcal{P}^*$ is injective in E_4 and E_5
Hence all the edge labelings in E_4 and E_5 are distinct.
Thus $\mathcal{P}^*$ is injective in E.
Hence the extended grid $EM(1,n)$ admits cube difference labeling, for $n \geq 2$.

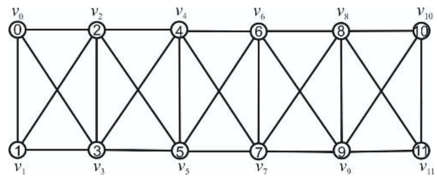

Figure. 1 cube difference labeling of the extended grid $EM(1,5)$

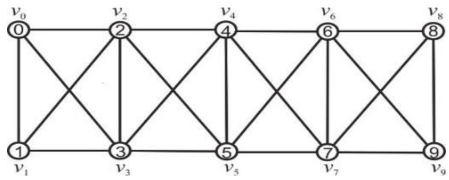

Figure. 2 cube difference labeling of the extended grid $EM(1,4)$

1. Conclusion

In this paper we prove the admittance of cube difference labeling of an extended grid $EM(1, n)$ for $n \geq 2$. Many graphs may admit cube difference labeling . An investigation to identify such graphs will be considered as future work.

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