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On Minuscule Topological Spaces

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Abstract

Numerous types of topological spaces are introduced every year by different topologists. In this article we introduced Minuscule topological spaces and discussed properties and applications. The present article discusses the aspects of a novel family of functions on M-topological spaces, referred to as minuscule topological spaces, in terms of of M-closed sets, M- closure and M -interior. Apart from that, a conscious effort is put forth to define M — *open* maps, M — *closed* maps and M — *homeomorphism*.

1. Introduction

Kelly (8) proposed bitopological spaces in 1963. Chang(5), in 1968, initially put forward the idea of fuzzy topological spaces, which were further investigated by Zadeh's (25) introduction of fuzzy sets. Mashhour et al. (11) first put forward the idea of Bisupra-topological spaces in 1983. Kuratowski first developed the concept of the Ideal in topological spaces. Furthermore, they defined local functions within the framework of ideal topological spaces. In 1990, Hamlett and Jankovic(6) looked into other aspects of topological space. One of the essential aspects of topology is the continuity of functions. Further development of the idea of nanotopology was done by Lellis Thivagar and Carmel Richard (21), expressed it in terms of approximations and the boundary region of a subset of the universe by imposing an equivalence relation on it. He provided the concepts of nano-closure, nano-interior, and nano-closed sets. Sakkraiveeranan Chandrasekar(4) utilized nano topology in 2019 to introduce microtopological spaces. A minimum of three nano open sets, such as U and Φ , and a maximum of five nano open sets are included in nano topology. By excluding boundary approximations and utilizing the symmetric difference, we can generate more open sets in nano topology. More open sets can be included due to this extension, generating what is known as minuscule topology. It's important to remember that not all micro topologies and nanotopologies belong to the minuscule topological space. In this study, we introduce a new class of functions defined on minuscule topological spaces called M-topological spaces. M-closed sets, M-closure, and M-interior characterizations have been utilized to describe their properties. We have also formalized the ideas of M-homeomorphisms, M- open maps, and M-closed maps, together with their representations with respect to M-closure and M-interior.

2. Preliminary

Let us now discuss the subsequent definitions, which will prove to be beneficial in the subsequent analysis.

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Definition 2.1. *[20] Consider a non-empty finite set* U *that consists of objects referred to as the universe. Let* E *be an equivalence relation on* U*, which is formally referred to as the indiscernibility relation. Subsequently, the set* U *is partitioned into disjoint equivalence classes. Elements that are part of the same equivalence class are considered to be indiscernible from each other. The pair* (U, E) *is widely referred to as the approximation space in research papers. Let* X *be a subset of* U.

1. *The lower approximation of the set* X *with respect to the relation* E *refers to the collection of objects that can be unambiguously categorized as belonging to* X *with respect to the conditions defined by* E*. This lower approximation can be expressed as* $L_E(X)$*. That is,* $L_E(X) = \bigcup_{x\in U}\{E(x) : E(x) \subseteq X\}$ *where* E(*x*) *denotes the equivalence class determined by x* ∈ U.

2. *The upper approximation of the set* X *in relation to the set* E *refers to the collection of all objects which have the chance to be considered as members of* X *in relation to* E*. This upper approximation is represented by the notation* $U_E(X)$*. That is* $U_E(X) = \bigcup_{x\in U}\{E(x) : E(x) \cap X \neq \Phi\}$

3. *The boundary region of the set* X *in relation to the reference set* E *is defined as the collection of every object that cannot be definitively classified as either belonging to* X *or not belonging to* X *with respect to* E*. This boundary region is usually referred to as* $B_E(X)$*. That is,* $B_E(X) = U_E(X) - L_E(X)$.

Definition 2.2. *[20] Let* U *be an arbitrary universe, and let* E *represent an equiv- alence relation defined on* U*. and* $\tau_E(X) = \{U, \Phi, L_E(X), U_E(X), B_E(X)\}$ *.and let* X *be a subset of* U *that adheres to the subsequent axioms.*

1. $U, \Phi \in \tau_E(X)$

2. *The union of the elements of any sub-collection of* $\tau_E(X)$ *is in* $\tau_E(X)$.

3. *The intersection of the elements of any finite sub collection of* $\tau_E(X)$ *is in* $\tau_E(X)$*. The nano topology on the set* U *with respect to* X *is denoted as* $\tau_E(X)$*. The topological space* $(U, \tau_E(X))$ *is referred to as the nano topological space. The individual elements of a set are collectively referred to as nano open sets.*

Example 2.3. *Let* $U = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ *with* $U/E = \{\{\xi_1\}, \{\xi_3\}, \{\xi_2, \xi_4\}\}$.

Let $X = \{\xi_1, \xi_2\} \subseteq U$*, Then* $\tau_E(X) = \{U, \Phi, \{\xi_4\}, \{\xi_1, \xi_2, \xi_4\}, \{\xi_2, \xi_4\}\}$.

Definition 2.4. *[4] In the context of nano topological spaces, let* $(U, \tau_E(X))$ *repre- sent a particular instance. In this case, we define the micro topology of* $\tau_E(X)$ *as*

$\mu_E(X) = \{N \cup (N' \cap \mu)\}$*, where* N *and* N′ *are elements of* $\tau_E(X)$ *and* μ *is a set not belonging to* $\tau_E(X)$.

Definition 2.5. *[4] The Micro topology, denoted as* $\mu_E(X)$*, adheres to the follow- ing axioms.*

1. $U, \Phi \in \mu_E(X)$

2. *The union of the elements of any sub-collection of* $\mu_E(X)$ *is in* $\mu_E(X)$.

3. *The intersection of the elements of any finite sub collection of* $\mu_E(X)$ *is in* $\mu_E(X)$.

The Micro topology on U *with respect to* X *is denoted as* $\mu_E(X)$*. The mathematical structure denoted by the triplet* $(U, \tau_E(X), \mu_E(X))$ *is referred to as a micro topolog- ical space. Within this context, the components of* $\mu_E(X)$ *are commonly known as micro open sets, whereas the counterpart of a micro open set is referred to as a micro closed set.*

Example 2.6. *Let* $U = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ *with* $U/E = \{\{\xi_1\}, \{\xi_3\}, \{\xi_2, \xi_4\}\}$.

Let $X = \{\xi_1, \xi_2\} \subseteq U$*, Then* $\mu_E(X) = \{U, \Phi, \{\xi_1\}, \{\xi_3\}, \{\xi_2, \xi_4\}, \{\xi_1, \xi_3\}, \{\xi_2, \xi_3, \xi_4\}, \{\xi_1, \xi_2, \xi_4\}\}$.

3. Minuscule Topological Spaces

Definition 3.1. *Consider a non-empty finite set* U *that consists of objects referred to as the universe. Let* E *be an equivalence relation on* U*, which is formally referred to as the indiscernibility relation.*

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Subsequently, the set U *is partitioned into disjoint equivalence classes. Elements that are part of the same equivalence class are considered to be indiscernible from each other. The pair* (U, E) *is widely referred to as the approximation space in research papers. Let* X *be a subset of* U.

1. *The lower approximation of the set* X *with respect to the relation* E *refers to the collection of objects that can be unambiguously categorized as belonging to* X *with respect to the conditions defined by* E. *This lower approximation can be expressed as* $L_E(X)$. *That is* $L_E(X) = \bigcup_{x \in U} \{E(X) : E(X) \subseteq X\}$ *where* E(X) *denotes the equivalence class determined by* X.
2. *The lower minimal approximation:* $L^{\Lambda}_E(X) = \bigcup_{x \in U} \{E(X) : E(X) \subseteq X\} - X = \overline{L_E(X) - X}$
3. *The upper approximation of* X *with respect to* E *is the set of all objects, which can be possibly classified as* X *with respect to* E *and it is denoted by* $U_E(X)$. *That is* $U_E(X) = \bigcup_{x \in U} \{E(X) : E(X) \cap X \neq \Phi\}$
4. *The upper minimal approximation:* $U^{\Lambda}_E(X) = \bigcup_{x \in U} \{E(X) : E(X) \cap X \neq \Phi\} - X = \overline{U_E(X) - X}$.
5. *Let* $L_E(X)$ *and* $U_E(X)$ *be two sets. The symmetric difference of the sets* $L_E(X)$ *and* $U^{\Lambda}_E(X)$ *is* $L_E(X) \Delta U^{\Lambda}_E(X)$ *and it is denoted by* $L_E(X) \nabla U^{\Lambda}_E(X) = (L_E(X) - U^{\Lambda}_E(X)) \cup (U^{\Lambda}_E(X) - L_E(X))$

Definition 3.2. *Let* U *be the universe,* E *be an equivalence relation* U *and*

$\lambda_E(X) = \{U, \Phi, L_E(X), U_E(X), L^{\Lambda}_E(X), U^{\Lambda}_E(X), L_E(X) \nabla U^{\Lambda}_E(X)\}$. *Here* $L^{\Lambda}_E(X)$ *is always be* Φ. Φ *is always belongs to the topology. So,* $L^{\Lambda}_E(X)$ *and is neglected.*

Then the topology $\lambda_E(X) = \{U, \Phi, L_E(X), U_E(X), U^{\Lambda}_E(X), L_E(X) \nabla U^{\Lambda}_E(X)\}$ *where* $X \subseteq E$.

$\lambda_E(X)$ *satisfies the following axioms:*

1. U *and* $\Phi \in \lambda_E(X)$.
2. *The union of the elements of any sub collection of* $\lambda_E(X)$ *is in* $\lambda_E(X)$.
3. *The intersection of all elements of any finite sub collection of* $\lambda_E(X)$ *is in* $\lambda_E(X)$. *That is,* $\lambda_E(X)$ *is a topology on* U *called the* M - *topology on* U *with respect to* X. *We call* $(U, \lambda_E(X))$ *is a Minuscule topological space (or)* M - *topological space*

. *The elements of* $\lambda_E(X)$ *are called* M - *open sets.*

Properties: If (U, E) is an approximation space and X, W ⊆ U, then

1. $L_E(X) \subseteq X \subseteq U_E(X)$.
2. $L_E(\Phi) = U_E(\Phi) = \Phi$ and $L_E(X) = U_E(X) = U$
3. $U_E(X \cup W) = U_E(X) \cup U_E(W)$
4. $U_E(X \cap W) \subseteq U_E(X) \cap U_E(W)$
5. $L_E(X \cup W) \supseteq L_E(X) \cup L_E(W)$
6. $L_E(X \cap W) = L_E(X) \cap L_E(W)$
7. $L_E(X) \subseteq L_E(W)$ and $U_E(X) \subseteq U_E(W)$ whenever $X \subseteq W$
8. $L^{\Lambda}_E(X) = \Phi$
9. $L^{\Lambda}_E(X) \cap U_E(X) = \Phi$
10. $U^{\Lambda}_E(X \cup W) \subseteq U^{\Lambda}_E(X) \cup U^{\Lambda}_E(W)$
11. $U^{\Lambda}_E(X \cap W) = U^{\Lambda}_E(X) \cap U^{\Lambda}_E(W)$
12. $U_E(U^{\Lambda}_E(X)) = L_E(U^{\Lambda}_E(X)) = U^{\Lambda}_E(X)$.
13. $L_E(X) \nabla U_E(X) = U_E(X)$

Definition 3.3. *If* $(U, \lambda_E(X))$ *is a* M - *topological space ,where* X ⊆ U *and if* A ⊆ U, *The* M-*interior of set* A *can be defined as the union of all* M-*open subsets of* A, *denoted as* Mint(A). *The concept of*

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the M *-closure of a set A is defined as the intersection of all sets that are* M *-closed sets containing A. This* M*-closure is denoted by* M*cl*(*A*).

Remark 3.4. *The basis for the* M *-topology* λ_E (X) *with respect to X is given by*

$B_E(X) = \{U, L_E(X), L_E(X) \nabla U^{\Lambda}(X)\}$.

Example 3.5. *Let* $U = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ *with* $U/E = \{\{\xi_1\}, \{\xi_3\}, \{\xi_2, \xi_4\}\}$.

Let $X = \{\xi_1, \xi_2\} \subseteq U$, *Then* $\lambda_E(X) = \{U, \Phi, \{\xi_4\}, \{\xi_1, \xi_3, \xi_4\}, \{\xi_3\}, \{\xi_3, \xi_4\}\}$

Then the basis for the M − *topology is given by* $B_E(X) = \{U, \{\xi_4\}, \{\xi_3, \xi_4\}\}$

Definition 3.6. *Let* $(U, \lambda_E(X))$ *and* $(T, \lambda_{E'}(W))$ *be* M − *topological spaces. Then a mapping* $\Phi : (U, \lambda_E(X)) \to (T, \lambda_{E'}(W))$ *is* M − *continuous on* U *if the inverse image of every* M − *openset in* T *is* M − *open in* U.

Example 3.7. *Let* $U = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ *with* $U/E = \{\{\xi_1\}, \{\xi_2, \xi_3\}, \{\xi_4\}\}$

Let $X = \{\xi_1, \xi_3\} \subseteq U$ *Then,* $\lambda_E(X) = \{U, \Phi, \{\xi_1\}, \{\xi_1, \xi_2, \xi_3\}, \{\xi_2\}, \{\xi_1, \xi_2\}\}$ *and the* M − *closed sets in* U *are* $U, \Phi, \{\xi_2, \xi_3, \xi_4\}, \{\xi_4\}, \{\xi_1, \xi_3, \xi_4\}$, *and* $\{\xi_3, \xi_4\}$. *Let* $T = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ *with* $T/E' = \{\{\kappa_1\}, \{\kappa_2\}, \{\kappa_3, \kappa_4\}\}$ *Let* $W = \{\kappa_1, \kappa_3\} \subseteq T$ *Then,* $\lambda_{E'}(W) = \{T, \Phi, \{\kappa_1\}, \{\kappa_1, \kappa_3, \kappa_4\}, \{\kappa_4\}, \{\kappa_1, \kappa_4\}\}$ *and the* M − *closed sets in* T *are* $T, \Phi, \{\kappa_2, \kappa_3, \kappa_4\}, \{\kappa_2\}, \{\kappa_1, \kappa_2, \kappa_3\}$, *and* $\{\kappa_2, \kappa_3\}$. *Define a function* $f : U \to T$, *as* $f(\xi_1) = \kappa_2$, $f(\xi_2) = \kappa_4$, $f(\xi_3) = \kappa_3$, $f(\xi_4) = \kappa_1$. *Then* $f^{-1}(\{\kappa_1\}) = \{\xi_4\}$, $f^{-1}(\{\kappa_1, \kappa_3, \kappa_4\}) = \{\xi_1, \xi_3, \xi_4\}$ *and* $f^{-1}(\{\kappa_1, \kappa_4\}) = \{\xi_2, \xi_4\}$. *The* M *-continuity of* f *is proven by the conclusion that the inverse image of each* M *-open set in* T *is* M *-open in* U.*"*

In terms of M-closed set, the following theorem describes in M-continuous functions.

Theorem 3.8. *A function* $f : (U, \lambda_E(X)) \to (T, \lambda_{E'}(W))$ *is* M − *continuous if and only if the inverse image of every* M − *closed set in* T *is* M − *closed in* U.

Proof. Let f be M − *continuous* and F be M − *closed* in T. That is T − F is M − *open* in T. Since f is M − *continuous*, $f^{-1}(T - F)$ is M − *open* in U. That is $U - f^{-1}(F)$ is M − *open* in U. Therefore, $f^{-1}(F)$ is M − *closed* in U. Thus, the inverse image of every M − *closed* set in T is M − *closed* in U. if f is M − *continuous* on U. Conversely, let the inverse image of every M − *closed* set be M − *closed*. Let be M − *open* in T. Then T is M − *closed* in T. Then $f^{-1}(T - G)$ is M − *closed* in U. That is $U - f^{-1}(G)$ is M − *closed* in U. Therefore, $f^{-1}(G)$ is M − *open* in U. Thus, the inverse image of every M − *open* in T is M − *open* in U. That is f is M − *continuous* in U.

The subsequent theorem provides a description of M − *continuous* functions using M − *closure*.

Theorem 3.9. *A function* $f : (U, \lambda_E(X)) \to (T, \lambda_{E'}(W))$ *is* M − *continuous if and only if* $f(\mathrm{M}cl(Z)) \subseteq \mathrm{M}cl(f(Z))$ *for every subset* Z *of* U.

Proof. Let f be M − *continuous*

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and Z ⊆ U. Then f(Z) ⊆ T. Mcl(f(Z)) is M-*closed* in T. Since f is M-*continuous*, f^{-1}(Mcl(f(Z))) is M-*closed* in U. Since f(Z) ⊆ Mcl(f(Z)), Z ⊆ f^{-1}(Mcl(f(Z))). Thus, f^{-1}(Mcl(f(Z))) is M-*closed* set containing Z. But Mcl(Z) is the smallest M-*closed* set containing Z. Therefore, Mcl(Z) ⊆ f^{-1}(Mcl(f(Z))). That is, f(Mcl(Z)) ⊆ Mcl(f(Z)). Conversely, let f(Mcl(Z)) ⊆ Mcl(f(Z)) for every subset of Z of U. If F is M-*closed* in T. Since, f^{-1}(F) ⊆ U, f(Mcl(f^{-1}(F))) ⊆ Mcl(f(f^{-1}(F))) ⊆ Mcl(F). That is Mcl(f^{-1}(F)) ⊆ f^{-1}(Mcl(F)) = f^{-1}(F). Since F is M-*closed*. Thus Mcl(f^{-1}(F)) ⊆ f^{-1}(F). But f^{-1}(F) ⊆ Mcl(f^{-1}(F)). Therefore Mcl(f^{-1}(F)) = f^{-1}(F). Therefore, for each M-closed set F in T, f^{-1}(F) is M-closed in U. In this way, f is M-continuous.

Theorem 3.10. *Let* (U, ℷ$_E$(X)) *and* (T, ℷ$'_E$(W)) *be two* M-*topological spaces where* X ⊆ U *and* W ⊆ T. *Then* ℷ$'_E$(W) = {T, Φ, L$_E$(W), U$_E$(W), U$^\Lambda_E$(W), L$_E$(W)∇U$^\Lambda_E$(W)} *and its basis is given by,* B$'$ = {U, L$_E$(W), L$_E$(W)∇U$^\Lambda_E$(W)}.

A function f : (U, ℷ$_E$(X)) → *(T,* ℷ$'_E$(W)) *is* M-*continuous if and only if the inverse image of every member of* B$'$ *is* M-*open in* U.

Proof. Let f be M-*continuous* on U. Let B ∈ B$'$. Then B is M-*open* in T. That is B ∈ ℷ$'_E$(W). Since f is M-*continuous*, f^{-1}(B) ∈ ℷ$_E$(X). That is f^{-1}(B) is M-*open* in U. Conversely, let the inverse image of every member of B$'$ be open in U. Let G be a M-*open* in T. Then G = ⋃{B : B ∈ B$_1$} where B$_1$ ⊂ B$'_E$. Then f^{-1}(G) = f^{-1}(⋃{B : B ∈ B$_1$}) = ⋃{f^{-1}(B) : B ∈ B$_1$} where each f^{-1}(B) is M-*open* in U and hence their union, which is f^{-1}(G) is M-*open* in U. Thus f is M-continuous on U.

The above-mentioned theorem provides a characterization of M-continuous functions based on basis elements. In the subsequent theorem, we provide a characterization of functions that are M-continuous in terms of the inverse image of M-closure.

Theorem 3.11. *A function* f : (U, ℷ$_E$(X)) → (T, ℷ$'_E$(W)) *is* M-*continuous. if and only if* Mcl(f^{-1}(B)) ⊆ f^{-1}(Mcl(B)) *for every subset* B *of* T.

Proof. If f is M-*continuous* and B ⊆ T, f^{-1}(Mcl(B)) is M-*closed* in U and hence Therefore Mcl[f^{-1}(Mcl(B))] = f^{-1}(Mcl(B)). Since B ⊆ Mcl(B), f^{-1}(B) ⊆ f^{-1}(Mcl(B)),

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$Mcl(f^{-1}(B)) \subseteq Mcl(f^{-1}Mcl(B)) = f^{-1}(Mcl(B))$. That is $Mcl(f^{-1}(B)) \subseteq f^{-1}(Mcl(B))$. Conversely, let $Mcl(f^{-1}(B)) \subseteq f^{-1}(Mcl(B))$ for every $B \subseteq T$. Let B be M – *closed* in T. Then $Mcl(B) = B$. By assumption, $Mcl(f^{-1}(B)) \subseteq f^{-1}(Mcl(B)) = f^{-1}(B)$. Thus $Mcl(f^{-1}(B)) \subseteq f^{-1}(B)$, But $f^{-1}(B) \subseteq Mcl(f^{-1}(B))$. Therefore $Mcl(f^{-1}(B)) = f^{-1}(B)$ That is $f^{-1}(B)$ is M – *closed* in U for every M – *closed* set B in T. Therefore f is M – *continuous* on U.

The subsequent theorem presents a criterion for functions that are M – *continuous* by considering the inverse image of the M – *interior* of a subset of T.

Theorem 3.12. *A function* $f : (U, ℷ_E(X)) \to (T, ℷ_E(W))$ *is* M – *continuous on* U *if and only if* $f^{-1}(Mint(B)) \subseteq Mint(f^{-1}(B))$ *for every subset* B *of* T.

Proof. Let f be M – *continous* and $B \subset T$. Then $Mint(B)$ is M – *open* in $(T, ℷ_E(W))$. Therefore $f^{-1}(Mint(B))$ is M *open* in $(U, ℷ_E(X))$. $f^{-1}(Mint(B)) = Mint[f^{-1}(Min(B))]$. Also, $Mint(B) \subseteq B$ implies that $f^{-1}(M(int(B)) \subseteq f^{-1}(B)$. Therefore $Mint(f^{-1}(Mint(B))) \subseteq Mint(f^{-1}(B))$ That is $f^{-1}(Mint(B)) \subseteq Mint(f^{-1}(B))$. Conversely, let $f^{-1}(Mint(B)) \subseteq Mint(f^{-1}(B))$ for every subset B of T. If B is M – *open* in T. $Mint(B) = B$. Also, $f^{-1}(Mint(B)) \subseteq Mint(f^{-1}(B))$. That is $f^{-1}(B) \subseteq Mint(f^{-1}(B))$. But $Mint(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B) = Mint(f^{-1}(B))$. Thus $f^{-1}(B)$ is M – *open* in U for every M – *open* set B in T. Therefore f is M – *continuous*.

Definition 3.13. *A subset* Z *of a* M – *topological space* $(U, ℷ_E(X))$ *is said to be* M – *dense if* $Mcl(Z) = U$.

Remark 3.14. *Since* $Mcl(X) = U$ *defines a* M - *topological space* $(U, ℷ_E(X))$ *with respect to* X *where* $X \subset U$, *it follows that* X *is* M – *dense in* U.

Theorem 3.15. *Let* $f : (U, ℷ_E(X)) \to (T, ℷ_E(W))$ *be an onto,* M – *continuous function. If* Z *in* M – *dense in* U *then* $f(Z)$ *is* M – *dense in* T.

Proof. Since Z is M – *dense* in U, $Mcl(Z) = U$. Then $f(Mcl(Z)) = f(U) = T$. Since f is onto. Since f is M – *coninuous* on U. That is $f(Mcl(Z)) \subseteq Mcl(f(Z))$. Therefore $T \subseteq Mcl(f(Z))$. But $Mcl(f(Z)) \subseteq T$. Therefore $Mcl(f(Z)) = T$. In other words, $f(Z)$ is M – *dense* in T. Thus, a M – *continuous* function maps M – *dense* sets into M – □

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dense because it is surjective.

4. Minuscule-open map and Minuscule-closed map

Definition 4.1. *A function* f : (U, ℷE (X)) → (T, ג′ (W)) *is a* M—G *open map if the image of ever* M— *open set in* U *is* M—G *open in* T . *The mapping* f *is said to be a* M— *closed map if the image of every* M— *closed set in* U *is* M— *closed in* T .

Theorem 4.2. *A mapping* f : (U, ℷE(X)) → (T, ג′ (W)) *is* M— *closed map if and only if* M*cl*(f(Z)) ⊆ f(M*cl*(Z)) *for every subset* Z *of* U

Proof. If f is M— *closed*. f(M*cl*(Z)) is M— *closed* in T .since M*cl*(Z) ⊆ is M— closed in U. Since Z ⊆ M*cl*(Z) , f(Z) ⊆ f(M*cl*(Z)) . Thus f(M*cl*(Z)) is M— *closed* set containing f(Z) . Therefore M*cl*(f(Z)) ⊆ f(M*cl*(Z)) . Conversely, If M*cl*(f(Z)) ⊆ f(M*cl*(Z)) for every subset Z of

Theorem 4.3. *A mapping* f : (U, ℷE(X)) → (T, ג′ (W)) *is* M— *open map if and only if* f(M*int*(Z)) ⊆ M*int*(f(Z)) . *For every subset* Z U .

Proof is similar to that of the theorem 4.2

Definition 4.4. *A function* f : (U, ℷE(X)) → (T, ג′ (W)) *is said to be a* M— *homeomorphism if*

1. f *is one-to-one and onto*
2. f *is* M -*continuous and*
3. f *is* M -*open.*

Theorem 4.5. *Let* f : (U , ℷE(X)) → (T , ג′ (W)) *be a one-one onto mapping. Then* f *is* M— *homeomorphism if and only if* f *is* M— *closed and* M— *continuous*

Proof. Let f be a M— *homeomorphism* . Then f is M— *continuous* . Let F be an arbitrary M— *closed* set in (U, ℷE(X)) . Then U − F is M— *open* . Since f is M— *open* . f(U − F) is M— *open* in T . That is T − f(F) is M— *open* in T That is f(F) is M— *closed* in T . Thus , the image of every M— *closed* set in U is M— *closed* in T . That is , f is M— *closed*. Conversely, let f be M— *closed* and M— *continuous* . Let be M— *open* in (U , ℷE(X)) . Then U is M— *closed* in U . Since f is M— *closed*. f(U) = T f( ) is M— *closed* in T . Therefore, f( ) is M— *open* in T . Thus f is M— *open* and hence f is M— *homeomorphism*.

The subsequent theorem gives a condition on a function that is M— *continuous* under which equality holds in theorem : 3.5 .

Theorem 4.6. *A one-one map* f : (U, ℷE(X)) *onto* (T, ג′ (W)) *is a* M -*homeomorphism if and only if* f(M*cl*(Z)) = M*cl*(f(Z)) *for every subset* Z *of* U .

Proof. If f is a M— *homeomorphism*, f is M— *continuous* and M— *closed*. if Z ⊆ U, f(M*cl*(Z)) ⊂ M*cl*(f(Z)) . Since f is M— *continuous* . Since M*cl*(Z) is M— *closed* in U and f is M

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closed . f(M*cl*(Z)) is M closed in T . M*cl*(f(M*cl*(Z))) = f(M*cl*(Z)) . Since Z M*cl*f(Z) f(M*cl*(Z)) , and hence M*cl*(f(Z)) M*cl*(f(M*cl*(Z))) = f(M*cl*(Z)) . Therefore M*cl*(f(Z)) f(M*cl*(Z)) . Thus, f(M*cl*(Z)) = M*cl*(f(Z)) . If f is a M *homeomorphism*. Conversely,

If f(M*cl*(Z)) = M*cl*(f(Z)) for every subset Z of U. Then f is M *continous*. If Z is M closed in U. M*cl*(A) = Z, which implies f(M*cl*(Z)) = f(Z) . There- fore M*cl*(f(Z)) = f(Z) . Thus f(Z) is M *closed* in T for every M *closed* set Z in U . That is f is M *closed* . Also f is M *continuous* . Thus f is a M − *homeomorphism*.

[1] Application

In the following section, we use the idea of M *continuous* functions in a real-life situation.

Let us examine the relationship between the expense of a bike trip and the distance covered. Let U = {d_1, d_2, d_3, d_4, d_5, d_6 } relates to the universe of distances between a bus terminus and six distinct locations. and Consider the set T defined as T = {p,,r,s,t,u} , which represents the universe of bike fares required to visit the six venues in U from the bus terminus. It is realized that the forces are dependent upon a distance between the places. Let U/E = {{d_1},{d_2, d_3},{d_3,d_5},{d_6 }} and

Let X = {d_1, d_2, d_3} a subset of U. Then the M *topology* on U is given by

ℷE(X) = {U, Φ, {d_1 }, {d_1, d_2, d_3, d_4 , d_5 }, {d_4 , d_5}, {d_1, d_4, d_5} }. Let T/E = {{p},{q,s},{r,t},{u}} and W = {p,q,r} a subset of T . Then the M *topology*

ℷE(W) on T with respect to W is given by ℷE (W) = {T , Φ, {p}, {p,q, r,s, t}, {s, t}, {p, s, t}}. Define f : U → T as f(d_1) = *p*, f(d_2) = q, f(d_3) = r, f(d_4) = s, f(d_5) = t, f(d_6) = u . Then, f^{-1}(U) = T, f^{-1}(Φ) = Φ, f^{-1}({p}) = {d_1 }, f^{-1}({p, q, r, s, t}) = {d_1 , d_2 , d_3 , d_4 , d_5 }, f^{-1}({s, t}) = {d_4 , d_5 } and f^{-1}({p,s,t}) = {d_1, d_4, d_5}. This implies that the inverse image of every M-open set in T under the given mapping is also M-open in U . As a result, function f is regarded as M- continuous. Additionally, it is seen that the image of every M-open set in U is M open in T, and f is a bijective function. Therefore, the function f can be classified as a M-homeomorphism. Hence, the relationship between the cost of taking a bike trip and the distance traveled can be described as a M-homeomorphism.

[2] Conclusion

In the field of topology, researchers frequently introduce novel types of topological spaces on every year. This study focuses on the domain of Minuscule topological spaces, offering a comprehensive examination of their properties along with relevant practical applications.The concept of *Minuscule continuity* functions possesses a broad range of practical applications in many real-world contexts. Moreover, minuscule continuous functions exhibit an extensive variety of applications. In the near future, the use of minuscule continuous functions is expected to extend to a wider range of everyday scenarios.

[1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [12] [13] [14] [15] [**?**] [16] [18] [19] [21] [22] [11][23] [24] [25] [11] [17] [20]

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