



Heat Transfer Phenomenon for A Thermally Radiative Al₂O₃-Cu/ Water Based Hybrid Nanofluid Flow Past an Exponentially Stretched Porous Surface

¹K. Fatima, ²*T. Hymavathi

¹Department of Mathematics, Krishna University, Machilipatanam, AP, India.

²Department of Mathematics, AdikaviNannaya University, Rajamahendravaram, AP, India

*Corresponding Author: T.Hymavathi

(Received: 02 September 2023

Revised: 14 October

Accepted: 07 November)

KEYWORDS

Nanofluid,
Radiation
parameter, Hybrid
nanofluid,
Exponentially
stretching surface,
Porous Medium,
Keller Box Method

ABSTRACT:

This research investigates the fluid dynamics and heat transfer characteristics of a hybrid nanofluid. The nanofluid flows over an exponentially expanding surface while releasing heat during the process. Additionally, the study considers heat transport and radiation effects. By employing similarity variables to transform (PDEs) into (ODEs), we can obtain similarity solutions. An implicit Finite Difference approach called Keller Box Method is employed to compute precise solutions to the problem. To provide a visual representation of the significance of various characteristics, we present graphical representations considering the relevant physical parameters. A notable discovery of this research is the consistent enhancement in heat transfer rates observed in hybrid nanofluids compared to conventional fluids. This increase in heat transfer rate is primarily influenced by the thermal radiation parameter within hybrid nanofluids. We also depict velocity and temperature profiles graphically, which are influenced by factors such as porosity and radiation. We explore the impact of varying thermophysical properties on velocity and temperature distributions, with a particular focus on the radiation parameter and Prandtl number. These graphical tools serve as a means to illustrate and elucidate the research findings and conclusions.

I. INTRODUCTION

Nanofluids refer to solutions that combine nanoparticles with a carrier fluid. These nanoparticles can include metals like copper, aluminum, and silver, as well as nonmetals such as graphite, carbides like silicon carbides, nitrides like silicon nitride, or oxides like aluminum oxide. Common carrier fluids encountered in nanofluids are water, oil, and ethylene glycol. Base liquid with nanoparticles significantly enhances the thermal properties of the resulting mixture. Nanofluids concept was initially proposed by Choi in 1995, highlighting how nanoparticles in these fluids improve thermal conductivity and other heat transfer characteristics.

In high-temperature heat systems, radiation play a crucial role in controlling the flow of both liquid and heat. Without

the contribution of heat radiation, reliable hardware, satellites, nuclear power plants, gas turbines, and advanced transformation systems and weaponry would not be possible.

Thermal radiation is emitted due to the movement of particles within matter. This emission is a result of the heat generated by the motion of common atomic particles like electrons and protons, which is subsequently converted into electromagnetic radiation. All objects with temperatures above absolute zero emit thermodynamic radiation, with most emissions occurring in the infrared (IR) range under typical conditions. Electromagnetic radiation is generated when a moving charge accelerates or a dipole oscillates. Examples of thermal radiation include detectable using an infrared camera, and the radiation emitted by the cosmic microwave background.



In conclusion, non-Newtonian fluid mechanics has gained prominence in research over the past decade, thanks to numerous successes in experimental settings. The non-Newtonian fluid paradigm extends beyond nanofluids and encompasses various other industrial fluids, including biological liquids, motor oils, and polymeric liquids.

To create a hybrid nanofluid, the process involves suspending distinct types of nanoparticles within the base fluid. Hybrid nanofluids have demonstrated significant enhancements in heat transfer efficiency across a wide array of research fields. There is potential for even further improvement in heat transfer properties and reduced pumping power in forced convection applications through continued exploration within the field of thermal fluids. The following is a summary of the key and intriguing findings stemming from research on hybrid nanofluids as a medium for enhancing heat transfer. A particularly promising avenue of investigation is the utilization of hybrid nanofluids for heat transfer in porous media, as it may shed light on which characteristics have an impact on the rate of heat transfer.

The field of heat transfer inside nanofluids has been significantly advanced by the notable research efforts of S.K. Das et al. [1]. Their contributions have established a strong basis for future investigations in this area. The intriguing experiment done by Oztup and Abu-Nada [2] focused on the phenomenon of spontaneous convection in nanofluids inside partly heated rectangular enclosures. The study conducted by Sajid et al. [3] investigated the effects of thermal radiation on the flow of a boundary layer and heat transfer in a viscous fluid across an exponentially stretched sheet. In their study, Gbadeyan et al. [4] performed a numerical analysis to investigate the characteristics of boundary layer flow in a nanofluid. The study focused on the behaviour of a nanofluid near a linearly expanding sheet, taking into account the effects of heat radiation and an artificial magnetic field. The findings of this research yielded significant insights into the aforementioned flow phenomenon. Hymavathi et al. [5] conducted an investigation using a horizontal channel that included a heat source and suction to explore the effects of heat transfer on oscillatory flow in the context of magnetohydrodynamics (MHD). Nadeem and colleagues [6] discovered significant promise in the domains of heat

conduction and fluid movement. The authors, Usman et al. [7], conducted a study on the flow characteristics of a rotating Cu-Al₂O₃-water hybrid nanofluid across a three-dimensional stretched sheet. Their research highlighted the notable impacts of thermal conductivity and nonlinear thermal radiation. The study conducted by Hymavathi et al. [8] explored the phenomena of heat transfer and fluid flow in a non-Newtonian Casson fluid over a thermal radiation-heated exponentially expanding surface. The researchers presented compelling results from their investigation.

Additionally, Ghadikolaei and colleagues [9] provide valuable insights into the influence of nonlinear thermal radiation and the Joule heating effect on the flow of magneto Casson nanofluids across an inclined porous stretched sheet. In a study conducted by Jawad [10], the researcher examined the impact of thermal radiation and velocity slip on a stretched sheet that is being heated by convection. Additionally, the study explored the impacts of magnetohydrodynamics (MHD) in the vicinity of the stagnation point. In their study, Waini et al. [11] highlighted the significant capacity of heat transmission and dynamic flow in hybrid nanofluids when applied to a stretching/shrinking sheet. In a separate study, Prakash et al. [12] investigated the hydromagnetic flow behaviour of a Casson fluid across a vertically oscillating plate subjected to a perpendicular magnetic field. The study included the effects of thermal radiation and a heat source. In their study, Jamaludin et al. [13] investigated the characteristics of mixed convection stagnation-point flow and heat transfer over a permeable stretching/shrinking sheet. The researchers examined the impact of the magnetic field and heat sources/sinks, while also exploring the potential use of a Cu-Al₂O₃/Water hybrid nanofluid for this particular problem. In their study, Hymavathi [14] examined the characteristics of magnetohydrodynamic (MHD) flow and heat transfer in a Casson fluid on a stretched surface with enhanced permeability. The mobility of an MHD Casson fluid on a flat plate inside a non-Darcy porous environment was investigated by Ganesh et al. [15]. The study conducted by Vyakarnam et al. [16] included a thorough examination of the properties of MHD Casson non-Newtonian nanofluids. Additionally, the researchers assessed the impact of heat radiation and chemical interaction on a nonlinear, permeable,



extended sheet. The study conducted by Wahid et al. [17] investigated the characteristics of flow and heat transmission in a hybrid nanofluid across a curved surface that experiences exponential stretching or shrinking. In their research, Ghasemi et al. [18] undertook a study aimed at providing a comprehensive understanding of the impact of solar radiation on the flow characteristics and heat transfer process of a nanofluid across a stretched sheet. The researchers Jawad et al. [19] used a hybrid nanofluid in their study to examine the phenomenon of stagnation point flow with melting heat transfer across a stretched surface. In their study, Alshuhail et al. [20] did an extensive literature review that specifically examined the use of mononad hybrid nanofluids in order to improve the efficiency of solar thermal systems. In their study, Hymavathi et al. [21] used computational techniques to examine the movement of Williamson nanofluids over a porous surface that undergoes exponential stretching, resulting in the emission of heat. In their study, Zainal et al. [22] investigated the impact of Arrhenius kinetics and radiation on the flow of a hybrid nanofluid across a stretching/shrinking sheet near the stagnation point.

II. MATHEMATICAL FORMULATION

Here, we consider a thermally radiative hybrid nanofluid moving steadily across an exponentially stretched porous surface in two dimensions. The temperature distribution is assumed to be $T_w = T + T_0 e^{x/L}$, where T_0 is the reference temperature, the fluid velocity is assumed to be $U = U_0 e^{x/L}$, and the sheet is considered to be stretched exponentially along the x-axis.

The elementary equations are articulated as follows

$$u_x + v_y = 0 \quad \rightarrow (1)$$

$$u u_x + v u_y = \frac{\mu_{hnf}}{\rho_{hnf}} u_{yy} - \vartheta \frac{u}{k_1} \quad \rightarrow (2)$$

$$u T_x + v T_y = \frac{K_{hnf}}{(\rho C_p)_{hnf}} T_{yy} - \frac{1}{(\rho C_p)_{hnf}} (q_r)_y \quad \rightarrow (3) \quad \text{III.}$$

Boundary Conditions

The following are the boundary conditions for the following problem

$$\left. \begin{aligned} u = U = U_0 e^{x/L}, v = 0, T = T_w \text{ at } y = 0 \\ u \rightarrow 0, T \rightarrow T_\infty, \text{ as } y \rightarrow \infty \end{aligned} \right\} \rightarrow (4)$$

Where (u, v) stands for the velocities of the hybrid nanofluid along the axes, which opens up fascinating possibilities for further research and deeper comprehension. In Table 1[3], we see the fascinating thermophysical properties of the hybrid nanofluid, and in Table 2[3], we see the useful correlations. Here, ϕ_1 and ϕ_2 stand for the exciting nanoparticle volume percentages of alumina and copper, respectively; C_p is shorthand for the remarkable specific heat capacity at constant pressure. The exceptional heat capacity, density, and thermal conductivity values of C_p , ρ , and k are also available. The remarkable alumina nanoparticle, copper nanoparticle, and base fluid represented by the symbols s_1 , s_2 , and f should not be forgotten.

The Rosseland approximation for the radiative heat flow, q_r , looks like this:

$$q_r = - \frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial y}$$

Where Stefan-Boltzman constant (σ) and absorption coefficient k^* are two variables.

Considering Temperature gradients can represent T^4 as a function of temperature. We can derive the following expression:

$$T^4 = 4T_\infty^3 - 3T_\infty^4$$

Then equation (3) becomes

$$u T_x + v T_y = \frac{K_{hnf}}{(\rho C_p)_{hnf}} T_{yy} - \frac{16\sigma T_\infty^3}{3(\rho C_p)_{hnf} k^*} T_{yy} \rightarrow (5)$$

Solution

For boundary conditions (4), we utilize the similarity transformations below to find similarity solutions for equations (1) - (3).



$$\left. \begin{aligned} \eta &= \sqrt{\frac{U_0}{2\nu L}} e^{x/2l} y, u = U_0 e^{x/l} f'(\eta) \\ v &= -\sqrt{\frac{\nu U_0}{2L}} e^{x/2l} [f(\eta) + f'(\eta)] \\ T &= T_\infty + T_0 e^{x/2l} \theta(\eta) \end{aligned} \right\}$$

→ (6)

Table 1:

The Thermophysical properties of fluid and nanoparticles:

Physical Properties	Water(f)	Alumina(Al ₂ O ₃) (s ₁)	Copper(Cu) (s ₂)
C _p (J/Kg K)	4179	765	385
ρ(kg/m ³)	997.1	3970	8933
K(W/mK)	0.613	40	400

Table 2:

The Thermophysical properties of nanofluid and hybrid nanofluid:

Properties	Nanofluid	Hybrid Nanofluid
Density	$\rho_{nf} = (1 - \phi_1) \rho_f + \phi_1 \rho_{s1}$	$\rho_{hnf} = (1 - \phi_2)[(1 - \phi_1)\rho_f + \phi_1 \rho_{s1}] + \phi_2 \rho_{s2}$
Heat Capacity	$(\rho C_p)_f + \phi_1 (\rho C_p)_{s1}$	$[(1 - \phi_1) (\rho C_p)_f + \phi_1 (\rho C_p)_{s1}] + \phi_2 (\rho C_p)_{s2}$
Dynamic viscosity	$\mu_{nf} = \frac{\mu_f}{(1 - \phi_1)^{2.5}}$	$\mu_{hnf} = \frac{\mu_f}{(1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5}}$
Thermal Conductivity	$k_{nf} = \frac{k_{s1} + 2k_f - 2\phi_1(k_f - k_{s1})}{k_{s1} + 2k_f + \phi_1(k_f - k_{s1})} * (k_f)$	$k_{hnf} = \frac{k_{s2} + 2k_{nf} - 2\phi_2(k_{nf} - k_{s2})}{k_{s2} + 2k_{nf} + \phi_2(k_{nf} - k_{s2})} * (k_{nf})$

With the same similarity transformations, Equation (1) is satisfied, and Equations (2) and (5) are simplified to a non-linear ODE as

$$f''' + \left((1 - \phi_2)[(1 - \phi_1) + \phi_1 \frac{\rho_{s1}}{\rho_f}] + \phi_2 \frac{\rho_{s2}}{\rho_f} \right) ((1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5}) (ff'' - 2f'^2 - Kf') = 0$$

$$\left[1 + \frac{4Rd}{3k_h} \right] \theta'' + \left((1 - \phi_2) \left[(1 - \phi_1) + \phi_1 \frac{(\rho C_p)_{s1}}{(\rho C_p)_f} \right] + \phi_2 \frac{(\rho C_p)_{s2}}{(\rho C_p)_f} \right) \left(\frac{(\phi_1 k_{s1} + \phi_2 k_{s2}) + 2k_f - (\phi_1 k_{s1} + \phi_2 k_{s2}) + (\phi_1 + \phi_2)k_f}{(\phi_1 k_{s1} + \phi_2 k_{s2}) + 2k_f + 2(\phi_1 k_{s1} + \phi_2 k_{s2}) - 2(\phi_1 + \phi_2)k_f} \right) * Pr(f\theta' - f'\theta) = 0$$

$$f''' + D_1(ff'' - 2f'^2 - Kf') = 0 \quad \rightarrow (7)$$

$$\left[1 + \frac{4Rd}{3k_h} \right] \theta'' + D_2 Pr(f\theta' - f'\theta) = 0 \quad \rightarrow (8)$$

The transformed boundary conditions are as follows

$$f(0) = 0, f'(0) = 1, \theta(0) = 1 \text{ as } \eta = 0,$$

$$f(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad \rightarrow (9)$$

$$\text{Where } K = \text{porosity parameter} = \frac{2\theta l}{k_1 U_0} e^{-\frac{x}{l}}$$

$$\text{Pr} = \text{Prandtl number} = \frac{\vartheta(\rho C_p)_{nf}}{k_f}$$

$$D_1 = \left(\frac{\rho_{hnf}}{\rho_f} \right) = \left((1 - \phi_2)[(1 - \phi_1) + \phi_1 \frac{\rho_{s1}}{\rho_f}] + \phi_2 \frac{\rho_{s2}}{\rho_f} \right) * ((1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5})$$



$$D_2 = \left(\frac{(\rho Cp)_{hnf}}{(\rho Cp)_f} \right)$$

$$\frac{(\rho Cp)_{hnf}}{(\rho Cp)_f} = \left((1 - \phi_2) \left[(1 - \phi_1) + \phi_1 \frac{(\rho Cp)_{s1}}{(\rho Cp)_f} \right] + \phi_2 \frac{(\rho Cp)_{s2}}{(\rho Cp)_f} \right),$$

$$k_h = \frac{k_{hnf}}{k_f} = \left(\frac{\left(\frac{\phi_1 k_{s1} + \phi_2 k_{s2}}{\phi_1 + \phi_2} \right) + 2k_f + 2(\phi_1 k_{s1} + \phi_2 k_{s2}) - 2(\phi_1 + \phi_2)k_f}{\left(\frac{\phi_1 k_{s1} + \phi_2 k_{s2}}{\phi_1 + \phi_2} \right) + 2k_f - (\phi_1 k_{s1} + \phi_2 k_{s2}) + (\phi_1 + \phi_2)k_f} \right)$$

IV. NUMERICAL PROCEDURE

Given the prescribed boundary conditions, Cebeci and Bradshaw successfully solved the equations efficiently. The tasks listed below are prepared for completion.

1. We can produce a first order equation by utilizing the updated equations.
2. When creating difference equations, central differences are a useful tool.
3. The result is an algebraic equation that is created, which is an exciting result. It is then linearized using Newton's technique and presented in a beautiful way as a matrix vector.
4. One effective technique for resolving the linear system is the block tridiagonal elimination method.
5. Introduce

$$f' = p$$

$$p' = q$$

$$\theta' = t$$

equation (6), (7) are reduced to

$$\Rightarrow q' + D_1 (fq - 2p^2 - Kp) = 0$$

$$\Rightarrow t' + \frac{3k_h D_2 Pr}{3k_h + Rd} (ft - p\theta) = 0$$

Boundary conditions are expressed in terms of the variables of interest.

$$f(0) = 0, p(0) = 1, \theta = 1 \text{ at } \eta = 0$$

$$p(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

V. RESULTS AND DISCUSSIONS

Solutions for the current problem involving flow and heat transfer are examined in detail in this section. When used with Matlab, the Keller Box approach significantly improves numerical calculation.

The velocity and temperature curves are elegantly displayed in Figures 1 and 2, emphasizing the fascinating range of values for ϕ_1 and ϕ_2 . The graph shows that there is a positive association between the velocity of the hybrid nanofluid and the volumetric concentration of copper nanoparticles. Furthermore, the temperature profiles of the two fluids are negatively impacted by the volumetric concentration of nanoparticles.

Figures 3 and 4 are excellent examples of how changing the porosity parameters has a favorable effect. When the porosity parameter is changed, the velocity profiles for both fluids demonstrate a decline. That being said, it's excellent to observe that the volumetric concentration of nanoparticles is maintained at $\phi_1 = \phi_2 = 0.1$. On the contrary, the temperature profiles display a noticeable impact. Temperature distributions as a function of Prandtl number are elegantly displayed in Figure 5. The temperature profiles of hybrid and nanofluid both drop with increasing Pr. Radiation has an effect on temperature profiles, as seen in Figure 6, where higher radiation results in lower profiles. On the other hand, this shift may present intriguing prospects for future research and advancements.

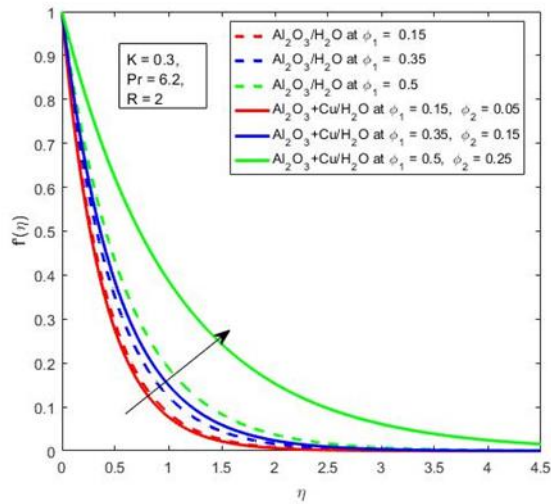


Figure- 1

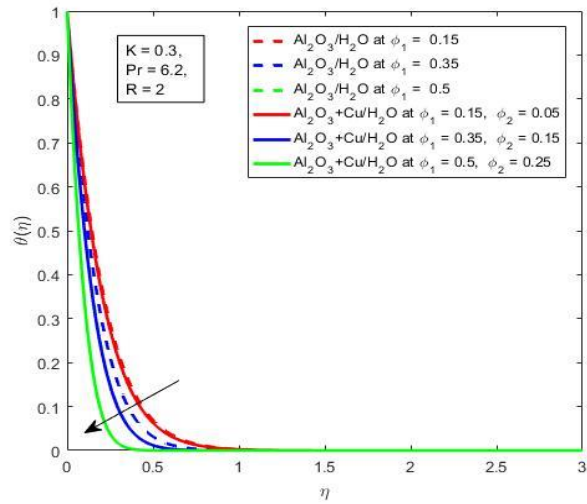


Figure 2

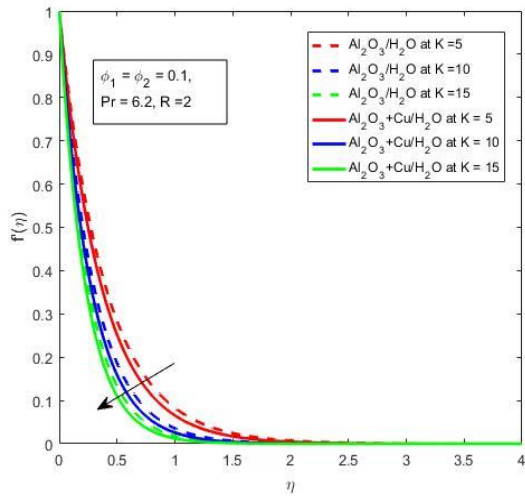


Figure- 3

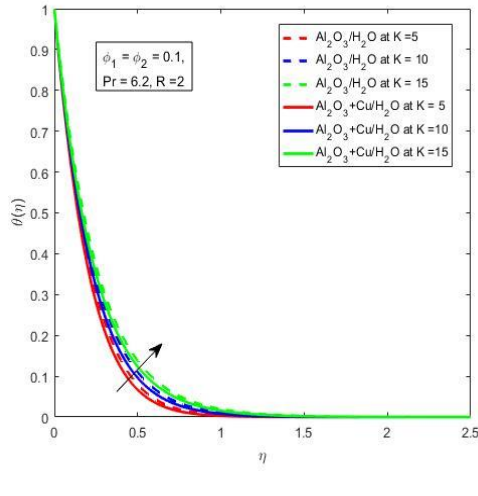


Figure- 4

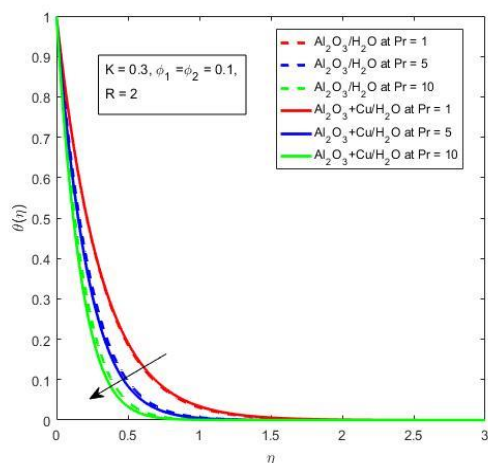


Figure- 5

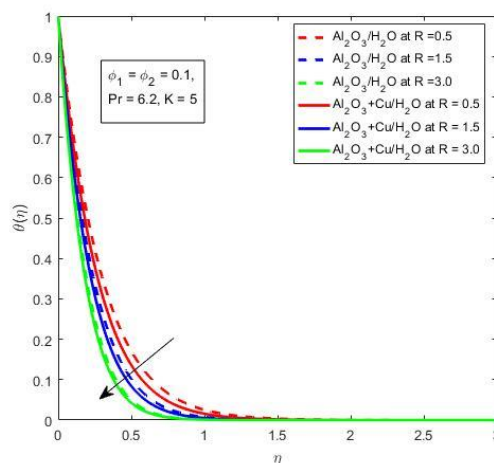


Figure- 6

References

- [1] S.K. Das, Stephen U.S. Choi and Hrishikesh E. Patel, Heat Transfer in Nanofluids – A Review, Heat Transfer Engineering, 27(10), 2006, 3-19.
- [2] HF.Oztup, E. Abu-Nada, Numerical Study of natural convection in partially heated rectangular enclosures filled with nanofluids, Int Journal of Heat Fluid Flow, 29, 2008,1326-1336.
- [3] M.Sajid, T.Hayat, Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet, International Communications in Heat and Mass Transfer, 35, 2008, 347-356.
- [4] Gbadeyan J. A and Olanrewaju P.O, Boundary layer Flow of a Nanofluid Past a Stretching Sheet with a Convective Boundary Condition in the Presence of Magnetic Field and Thermal Radiation, Australian Journal of Basic and Applied Sciences, 5(9), 2011, 1323-1334.
- [5] Hymavathi Talla, W.Sridhar, B.Sankar, N.N.V.Sakuntala, Heat Transfer to MHD Oscillatory Flow in a Horizontal Channel with Heat Source and Suction, International Journal of Math.Sci.&Engg.Appls, Vol.7(6), 2013, 85-95.
- [6] S. Nadeem and S.T. Hussain, Flow and Heat Transfer analysis of Williamson nanofluid, Appl Nanoscience, 4, 2014, 1005-1012.
- [7] M.Usman and W.Wang, Cu-Al₂O₃ /Water hybrid nanofluid through a permeable surface in the presence of nonlinear radiation and variable thermal conductivity via LSM, International Journal of Heat and Mass Transfer , 126,2018, 1347-1356.
- [8] T. Hymavathi, and W. Sridhar, Numerical Study of Flow and Heat Transfer of Casson Fluid Over an Exponentially Porous Stretching Surface in presence of Thermal Radiation, international Journal of Mechanical and Production Engineering Research and Development, 8(2018), 1145-1154.
- [9] S.S.Ghadikolaei, Kh. Hosseinzadeh, D.D.Ganji, B.Jafari, Nonlinear thermal radiation effect on magneto Casson nanofluid flow with Joule heating effect over an inclined porous stretching sheet, Case Studies in Thermal Engineering, 12, 2018, 176-187.
- [10] Jawad Raza, Thermal radiation and slip effects on magnetohydrodynamics(MHD) stagnation point flow of Casson fluid over a convective stretching sheet, Propulsion and Power Research, 8(2), 2019,138-146.
- [11] I.Waini and Ioan Pop, Unsteady flow and heat transfer past a stretching/shrinking sheet in a hybrid nanofluid, International Journal of Heat and Mass Transfer, 136(2019), 288-297.
- [12] G.Balaji Prakash and B.Mahboob, Thermal Radiation and Heat Source on MHD Casson Fluid overran Oscillating Vertical Porous Plate, Journal of



- Computer and Mathematical Sciences, 10(5), 2019, 1021-1031.
- [13] A.Jamaludin and Ioan Pop, MHD mixed convection stagnation-point flow of Cu- Al_2O_3 /water hybrid nanofluid over a permeable stretching/shrinking surface with heat source/sink, *European Journal of Mechanics/ B Fluids* 84, 2020, 71-80.
- [14] Hymavathi Talla, Numerical Study of MHD Flow and Heat Transfer over an Exponentially Permeable Stretching Surface of a Casson Fluid, *International Journal of Mechanical and Production Engineering Research and Development* 10(3), 2020, 2457-2464.
- [15] G.R.Ganesh and D.Sateesh Kumar, Forced Convection Heat Transfer of MHD Casson fluid in non Darcy Porous media, *International Journal of Advanced Science and Technology*, 29(6), 2020, 1313-1326.
- [16] S.Vyakarnam and Y.Hari Krishna, MHD Casson non-Newtonian nanofluid over a nonlinear penetrable elongated sheet with thermal radiation and chemical reaction, *Psychology and Education*, 58(1), 2021, 1776-1786.
- [17] N. Wahid and Mohd. Hafidzuddin, Flow and Heat Transfer of hybrid nanofluid induced by an exponentially stretching/shrinking curved surface, *Case Studies in Thermal Engineering*, 25(2021), 100982.
- [18] S.E. Ghasemi, M.Hatami, Solar Radiation effects on MHD stagnation point flow and heat transfer of a nanofluid over a stretching sheet, *Case Studies in Thermal Engineering*, 25, 2021, 1008988.
- [19] M.Jawad and Rashid Jan, Analysis of Hybrid Nanofluid Stagnation Point flow over a Stretching Surface with Melting Heat Transfer, *Hindawi, Mathematical Problems in Engineering*, <https://doi.org/10.1155/2022/9469164>.
- [20] L. Alshuhail and L.Syam Sundar, Thermal efficiency enhancement of mononad hybrid nanofluids in solar thermal applications – A review, *Alexandria Engineering Journal*, 68, 2023, 365- 404.
- [21] T.Hymavathi, K.Fatima and W.Sridhar, Numerical Approach to Thermally Radiative Williamson Nanofluid past an Exponentially Stretched porous Surface, *Corrosion and Protection*, 51(1), 2023, 979-986.
- [22] N.A.Zainal and Ioan Pop, Stagnation point hybrid nanofluid flow past a stretching/ shrinking sheet driven by Arrhenius kinetics and radiation effect, *Alexandria Engineering Journal*, 68, 2023, 29-38.