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Right-Censored Poisson Regression Model for Fertility Count Data

[1]Srinu Setti, [2]B.Muniswamy, [3]B.Punyavathi

[1,2,3]Department of Statistics, Andhra University, Visakhapatnam, Andhra Pradesh, India.

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KEYWORDS

fertility, count, caesarean-section delivery, right-censored Poisson, INLA

ABSTRACT:

Introduction: Count data represents the number of occurrences of an event within a fixed period [1, 2, 3, 4]. For example the number of caesarean-section deliveries in the lifetime of women. Count data is encountered in almost all research areas including economics, medicine, management, industrial organizations, and many more [5]. Count data is very common in various fields such as biomedical science, public health, and marketing. Poisson models are widely used in the regression analysis of count data and as a basis for count data analysis [6, 7, 8, 9, 10,11].

Objectives: The main aim of this study is to estimate the parameters of interest and compare the number of caesarean-section deliveries (NCSD) among women aged 15-49, in the state of Andhra Pradesh, India, using the right-censored Poisson regression model (RCPRM) and right-censored negative binomial regression model (RCNBRM). The fertility count data set, the real-world data of National Family Health Survey (NFHS-5), 2019-2021, from the Demographic and Health Surveys (DHS), 2019-2021 phase VII data is used for the analysis..

Methods: Investigating the delivery patterns among pregnant women. This study develops an algorithm based on Integrated Nested Laplace Approximation (INLA) for fitting the model NCSD in RCPRM and RCNBRM. The response variable NCSD is right-censored at 1, one caesarean-section delivery. The analysis is carried out using the INLA package in R.

Results: By use of the Deviance Information Criterion (DIC) and Watanabe-Akaike information criterion (WAIC), the result shows that the RCPRM; DIC (4467.14) and WAIC (4463.86) present a comparatively better fit in modelling the right-censored NCSD than the RCNBRM; DIC (4468.83) and WAIC (4465.08).

Conclusions: The INLA provides an efficient algorithm to model in RCPRM and RCNBRM. For further research, comparing the RCPRM with other models that estimate over-dispersion in count data is recommended.

1. Introduction

Count data represents the number of occurrences of an event within a fixed period [1, 2, 3, 4]. For example the number of caesarean-section deliveries in the lifetime of women. Count data is encountered in almost all research areas including economics, medicine, management, industrial organizations, and many more [5]. Count data is very common in various fields such as biomedical science, public health, and marketing. Poisson models are widely used in the regression analysis of count data and as a basis for count data analysis [6, 7, 8, 9, 10,11]. Poisson regression is one of the most popular techniques for the analysis of count data [10, 12, 13, 14]. Negative binomial regression is the extension of Poisson with a more liberal variance assumption and could collapse into Poisson regression with the dispersion parameter equal to 0 [15]. In real-life applications, count data often exhibits over-dispersion and excess zeroes. While Negative binomial regression can model count data with over-dispersion, both Hurdle [16, 17] and Zero-inflated [18, 19, 20, 21,22] regressions address the issue of excess zeroes in their rights. Over the past years, Poisson regression has been extended to accommodate censored count data. Although censoring is usually associated to lifetime data analysis, count data can also be censored,

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the most common type being right-censoring, which occurs when it is only known that the true count is higher than the observed one [23]. The Poisson regression model is extended to the censored case with a constant censoring threshold [24].

The fertility count data of NFHS-5 is used for modelling the NCSD. NFHS-5 can provide information on important indicators, which help track the progression of Sustainable Development Goals at various levels for SDG-1 "No Poverty", SDG-2 "Zero Hunger", SDG-3 "Good Health and Well-being" and SDG-5 "Gender Equality" [11, 25]. The NFHS-5 provides much-needed estimates on fertility, mortality, maternal, child, and adult health, women and child nutrition, etc. Most of these indicators highlight important aspects of family well-being in India. The NFHS-5 also provides information on several indicators covered in the Sustainable Development Goals (SDGs), which India is committed to. SDG-3, which says "Ensure healthy lives and promote well-being for all at all ages" in achieving the SDGs by 2030 (NFHS-5, 2019-2021) [11, 25].

2. Objectives

The main aim of this study is to estimate the parameters of interest and compare the number of caesarean-section deliveries (NCSD) among women aged 15-49, in the state of Andhra Pradesh, India, using the right-censored Poisson regression model (RCPRM) and right-censored negative binomial regression model (RCNBRM). The fertility count data set, the real-world data of National Family Health Survey (NFHS-5), 2019-2021, from the Demographic and Health Surveys (DHS), 2019-2021 phase VII data is used for the analysis.

3. Methods

3.1. Population and sample design: The National Family Health Survey 2019-21 (NFHS-5), the fifth in the NFHS series, provides information on population, health, and nutrition for India. NFHS-5 fieldwork for India was conducted in two phases— Phase-I from 17 June 2019 to 30 January 2020 covering 17 states and 5 union territories (UT), and Phase-II from 2 January 2020 to 30 April 2021 covering 11 states and 3 UTs—by 17 Field Agencies and interviews were completed with 724,115 women, gathered the information, for a response rate of 97 percent [11, 25].

3.2. Sample in the study: NFHS-5 fieldwork for Andhra Pradesh was conducted from 2nd July 2019 to 14 November 2019 by Sigma Research and Consulting Pvt. Ltd. Information was gathered from 10,975 women [11, 26]. The purposive sampling technique is used for the study. In the first stage, 724,115 women are considered. In the second stage of the purposive sampling method, 18,538 women from Andhra Pradesh were considered of which 7,563 women of Andhra Pradesh were interviewed in Phase-II.

In the final stage, 2,833 women aged 15-49 are considered by the purposive sampling technique. Births delivered by caesarean section in urban and rural is 50.5% and 39.3% respectively and the total is 42.4% for births in 5 years before the survey. Births in private health facilities that are delivered by caesarean section in urban and rural areas are 66.1% and 61.4% respectively and the total is 63.0% for births in 5 years before the survey. Births in public health facilities that are delivered by caesarean section in urban and rural are 30.9% and 25.2% respectively and the total is 26.6% for births in 5 years before the survey [11, 26].

3.3. Variables in the study: The following Table 1[11], briefs about the variables that are primarily and secondarily involved in the study of the number of caesarean section deliveries in Andhra Pradesh, India.

Table 1: Summary of variables considered for the study

Variable	Type	Value Description
The number of caesarean section delivery	Categorical	0 = "No caesarean section delivery", 1 = "One caesarean section delivery", 2 = "Two caesarean section deliveries"
During delivery, did you experience a breech presentation?	Categorical	0 = "No", 1 = "Yes", 2 = "Don't know"
Currently has heart disease	Categorical	0 = "No", 1 = "Yes", 2 = "Don't know"
High blood pressure	Categorical	0 = "No", 1 = "Yes", 2 = "Don't know"

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Prolonged labour	Categorical	0 = "No", 1 = "Yes", 2 = "Don't know"
Child is twin.	Categorical	0 = "Single birth", 1 = "1st of multiple", 2 = "2nd of multiple", 3 = "3rd of multiple", 4 = "4th of multiple", 5 = "5th of multiple"
Respondent's current age	Interval	15,16,17,….., 49
Highest educational level	Categorical	0 = "No education", 1 = "Primary", 2 = "Secondary", 3 = "Higher"

Where NCSD is transformed by combining variables of last birth a caesarean section and delivery by caesarean section, high blood pressure is transformed by combining variables told about pregnancy complication: high blood pressure and told had high BP on two or more occasions by a doctor or other health professional and prolonged labour is transformed by combining variables told about pregnancy complication: prolonged labour and during delivery, did you experience prolonged labour?. And the missing values are replaced with "2 = Do not know". The response variable is censored on the right at threshold [24, 27, 28] 1, one caesarean-section delivery. The model is: Right-censored NCSD = S441 + S728E + HBP + PL + B0 + V012 + V106

The right-censored number of caesarean section deliveries = During delivery, did you experience a breech presentation? + Currently has heart disease + High blood pressure + Prolonged labour + Child is twin + Respondent's current age + Highest educational level. The explained variable the right-censored number of caesarean section deliveries is defined as "0", "1" and "2" or more NCSD. The explanatory variables are during delivery, did you experience a breech presentation?, currently has heart disease, high blood pressure, prolonged labour, the child is twin, respondent's current age, and highest educational level.

The count data outcome variable the number of caesarean-section deliveries is right-censored at 1, one caesarean-section delivery, modelled to fit the right-censored Poisson regression model [7, 24, 27, 29]. The model is fitted with RCPRM and then with RCNBRM. If the mean is greater than the variance then it is said to be over-dispersed [30] otherwise it is said to be under-dispersed. The Integrated Nested Laplace Approximation (INLA) [1, 11, 31, 32, 33, 34, 35, 36, 37] is used for estimating unknown parameters in the RCPRM. The INLA methodology ensures computational efficiency by using sparse representations of high dimensional matrices used in Latent Gaussian Models (LGMs) [31, 32, 38, 39].

3.4. Statistical Model and Parameter Estimation: The general mathematical form of RCPRM is $log(y) = \alpha + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p$ where y is the right-censored explained variable, α and β are numeric coefficients; α being the intercept, sometimes α also is represented by β_0, it's the same and x is the explanatory variable. Consider an equation with seven predictor variables and one predictand variable:

$\log(y) = \alpha + \beta_p x_p$, where $p = 1,2,\ldots,7$ ------------ 1

$\log(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7$ -------------------------------------- 2

where y = number of caesarean section delivery, x_1 = during delivery, did you experience a breech presentation? x_2 = currently has heart disease, x_3 = high blood pressure, x_4= prolonged labour, x_5= child is twin, x_6= respondent's current age and x_7 = highest educational level. This is equivalent to: $y = e^{(\alpha+ \beta(x))} = e^{\beta_0} + e^{\beta_p * x_p}$ ------------------------------ 3

The negative binomial distribution is a function of both mean (μ) and alpha (α); the dispersion parameter, as α →0; the distribution becomes the Poisson distribution [15, 37]. The form of the model equation for NBRM is the PRM. The log of the outcome is predicted with a linear combination of predictors [40]. If α →0, then the censored negative binomial (CNB) distribution converges to the censored Poisson (CP) distribution [41]. Similarly if α →0, then the RCNB distribution converges to the RCP distribution. Hence equations 1, 2, and 3 are obtained for RCNBRM. Then from equation 3, the following equations are

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$$y = e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7)} \text{ ---- } 4$$

$$y = e^{\beta_0} * e^{\beta_1 x_1} * e^{\beta_2 x_2} * e^{\beta_3 x_3} * e^{\beta_4 x_4} * e^{\beta_5 x_5} * e^{\beta_6 x_6} * e^{\beta_7 x_7} \text{ -- } 5$$

4. Results

4.1. Descriptive Statistics: The following are a few tables that explain the descriptive Statistics of the respondents:

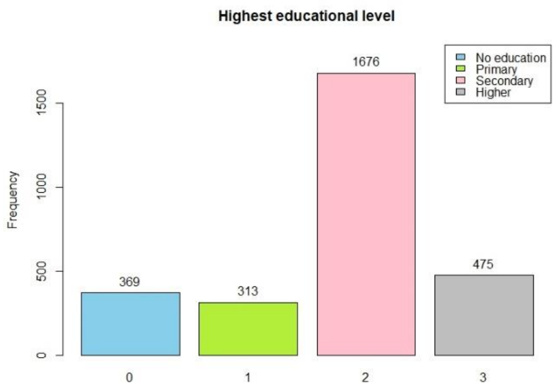

Figure 1: Highest educational level

Figure 1, the highest educational level of the respondents describes that 1676 respondents have secondary education as their highest educational level, followed by 475 respondents who have higher education, then 369 respondents have no education and 313 respondents have primary education.

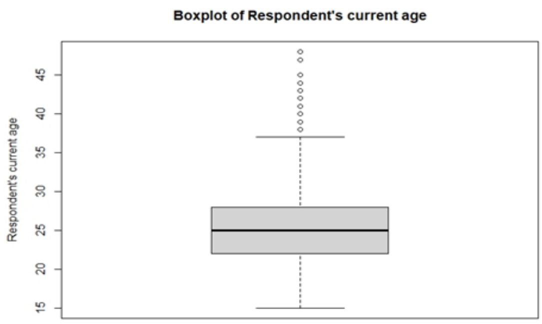

Figure 2: Respondent's current age

Figure 2 is the boxplot of the respondent's current age. The boxplot shows that the respondent's current age is centred at 25 years old.

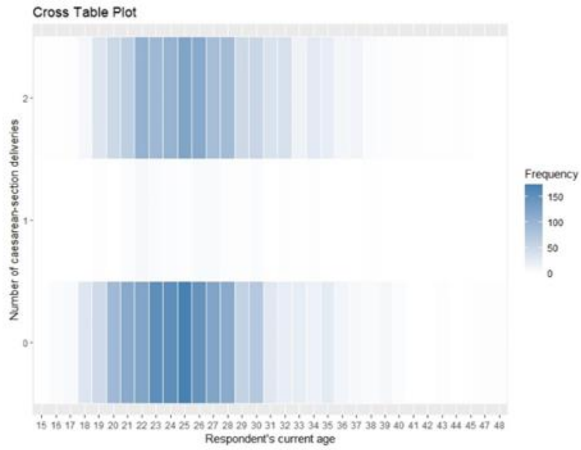

Figure 3: Cross table plot

Figure 3 explains the cross-tabulation of respondent's current age and the NCSD model. It shows that the highest number of women, aged 25, have 2 NCSD, followed by aged 26 years, women have 2 NCSD.

4.2. Women's socio-demography findings in the fitted model: Below are a few figures that show women's socio-demography findings:

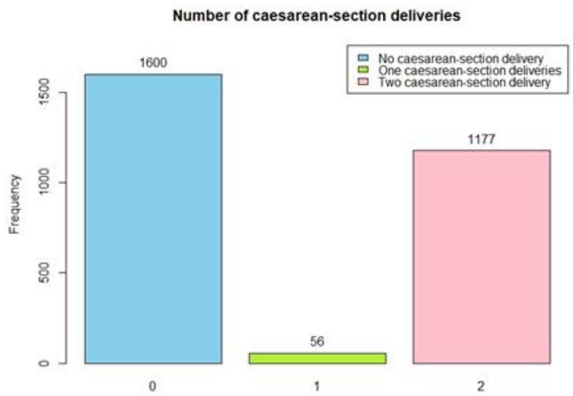

Figure 4: Number of caesarean-section deliveries

Figure 4 gives the details that 1600 respondents who have no caesarean section delivery, followed by 1177 respondents who have 2 caesarean section deliveries and 56 respondents who have 1 caesarean section delivery.

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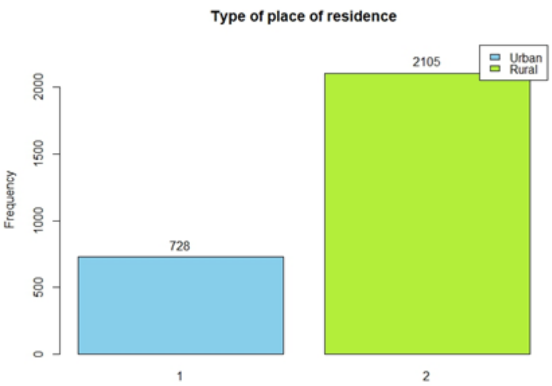

Figure 5: Type of place of residence

Figure 5 describes that the majority of 2105 respondents are from rural areas. A few 728 respondents are from urban areas.

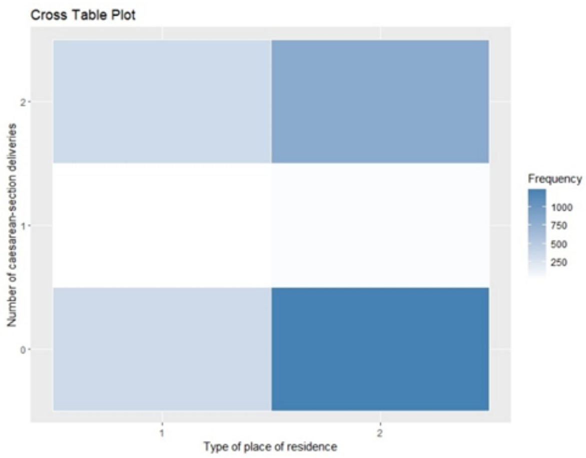

Figure 6: Cross table plot

Figure 6 shows the cross-tabulation of the type of place of residence and the number of caesarean section deliveries. For most women in rural areas have 2 caesarean section deliveries. Fewer women in urban areas have 2 caesarean section deliveries.

4.3. Models Comparisons Criteria: The model right-censored number of caesarean section deliveries is fitted in RCPRM using INLA, fixed effects. The result is as follows:

Table 2: Summary of values of fixed effects and model hyperparameters of RCPRM

Fixed effects:	Mean	Standard deviation	0.025quant	0.5quant	0.975quant	Mode	KLD
(Intercept)	-1.8	0.187	-2.17	-1.8	-1.44	-1.8	0
During delivery, did you experience a breech presentation?	0.04	0.042	-0.04	0.04	0.126	0.04	0
Currently has heart disease	-1.2	0.708	-2.58	-1.2	0.193	-1.2	0
High blood pressure	0.01	0.058	-0.1	0.01	0.125	0.01	0
Prolonged labour	-0.1	0.07	-0.23	-0.1	0.045	-0.09	0
Child is twin	0.05	0.128	-0.21	0.05	0.296	0.05	0
Respondent's current age	0.02	0.006	0.01	0.02	0.035	0.02	0
Highest educational level	0.24	0.035	0.17	0.24	0.309	0.24	0

Table 2 exhibits estimates of RCPRM. The mean or mode of the posterior distribution for each model parameter are determined, which are Bayesian parameter point estimates of the model [17].

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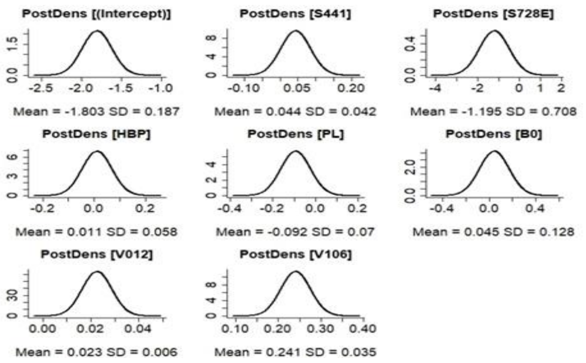

Figure 7: Fixed effects of RCPRM

The 'Fixed' effects of RCPRM represent a set of Posterior Densities (PostDens) from the posterior distribution as shown in Figure 7. The values 0’s of the Kullback-Leibler Divergence (KLD) indicate that the posterior distribution is well approximated by a Gaussian distribution. Through an estimate of the model's predictive accuracy, the best fit of a model is assessed. Cross-validation includes Akaike's Information Criterion (AIC) and Deviance Information Criterion (DIC) [42] when associated with Bayesian analyses. The DIC is a measure of the "goodness of fit" of a model penalizing for "complexity", and similar to AIC. The smaller the DIC better the model is. It can be used for comparing and ranking competing models.

Table 3: Summary of RCPRM values of marginal log-likelihood, DIC and WAIC

	MLIK	DIC	WAIC
RCPRM	-2272.93	4467.14	4463.86

Table 3 briefs the values of Marginal Log-Likelihood (MLIK), DIC and WAIC of RCPRM. The DIC of RCPRM is 4467.14. Wantanabe-Akaike Information Criterion or Widely Applicable Information Criterion (WAIC) [43], by contrast to AIC (and DIC) WAIC is a more fully Bayesian approach for estimating the out-of-sample expectation based on the log point-wise posterior predictive density. The WAIC of RCPRM is 4463.86. The MLIK of RCPRM is -2272.93.

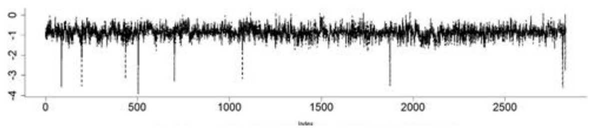

Figure 8.1: Linear predictor of RCPRM

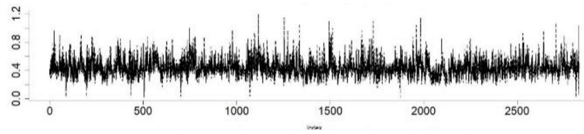

Figure 8.2: Fitted values of RCPRM

Figures 8.1 and 8.2 explains the linear predictor and fitted values of RCPRM respectively in posterior mean together with 0.025quant, 0.5quant and 0.975quant. Conditional Predictive Ordinate (CPO) is computed. The sum of the CPO values is a measure of fit. In RCPRM, there are no non-zero CPO values. Therefore none of the observations are unusual concerning the model. Probability Integral Transforms (PIT) provides a version of CPO that reveals whether or not any of the values are 'small' (all values must be between 0 and 1).

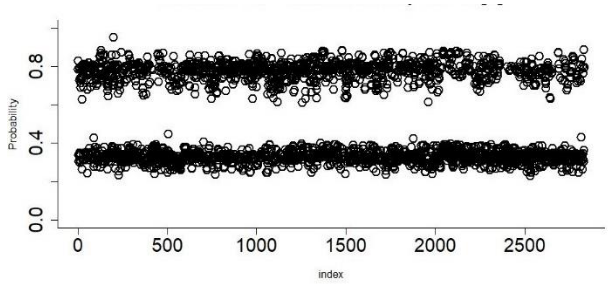

Figure 9.1: PIT values of RCPRM

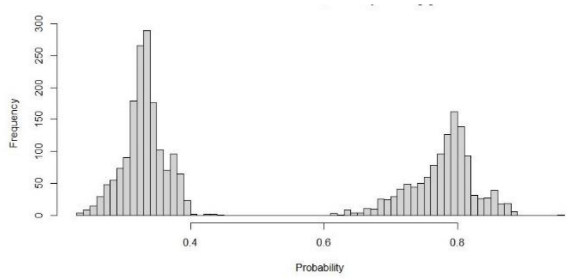

Figure 9.2: PIT values of RCPRM

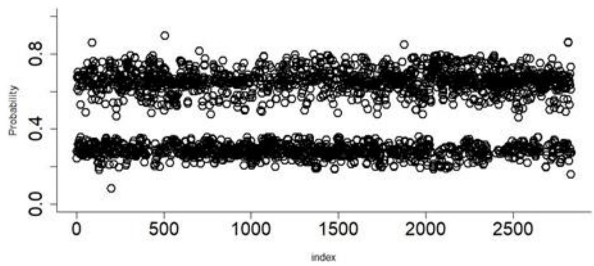

Figure 9.3: CPO values of RCPRM

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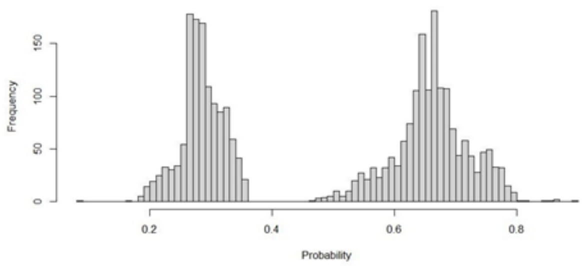

Figure 9.4: CPO values of RCPRM

Figures 9.1 to 9.4 briefs about the PIT and CPO values of RCPRM respectively. In RCPRM the PIT does not indicate a lack of fit for the values do not appear to deviate from a uniform distribution.

The model number of caesarean section delivery is fitted in RCNBRM using INLA, fixed effects. The result is as follows:

Table 4: Summary of values of fixed effects and model hyperparameters of RCNBRM

Fixed effects:	Mean	Standard deviation	0.025quant	0.5quant	0.975quant	Mode	KLD
(Intercept)	-1.803	0.187	-2.17	-1.803	-1.436	-1.8	0
During delivery, did you experience a breech presentation?	0.044	0.042	-0.04	0.044	0.126	0.044	0
Currently has heart disease	-1.194	0.708	-2.58	-1.194	0.194	-1.19	0
High blood pressure	0.011	0.058	-0.1	0.011	0.125	0.01	0
Prolonged labour	-0.092	0.070	-0.23	-0.092	0.045	-0.09	0
Child is twin	0.045	0.128	-0.21	0.045	0.296	0.05	0
Respondent's current age	0.023	0.006	0.01	0.023	0.035	0.02	0
Highest educational level	0.241	0.035	0.17	0.241	0.310	0.24	0
Model hyperparameters	98931.18	1166424.75	133	1684.21	448887.71	237	

The parameters of interest are estimated [17]. Table 4 portrays the estimation of the parameters of RCNBRM.

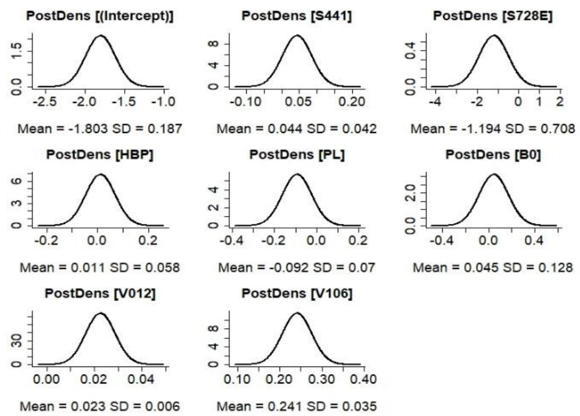

Figure 10: Fixed effects of RCNBRM

Figure 10 showcases the 'Fixed' effects [44] of RCNBRM, representing a set of PostDens from the posterior distribution. The KLD values “0” of RCNBRM indicate that the posterior distribution is well approximated by a Gaussian distribution.

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Table 5: Summary of RCNBRM values of marginal log-likelihood, DIC and WAIC

	MLIK	DIC	WAIC
RCNBRM	-2274.29	4468.83	4465.08

Table 5 shows clearly the values of MLIK, DIC, and WAIC of RCNBRM. The DIC of RCNBRM is 4468.83 and WAIC is 4465.08. The MLIK of RCNBRM is -2274.29.

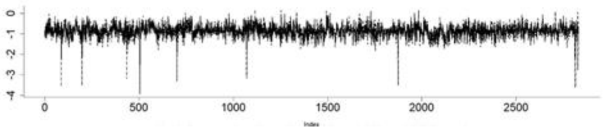

Figure 11.1: Linear predictor of RCNBRM

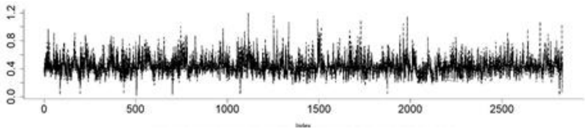

Figure 11.2: Fitted values of RCNBRM

Figures 11.1 and 11.2 explains the linear predictor and fitted values of RCNBRM respectively in posterior mean together with 0.025quant, 0.5quant and 0.975quant. In RCNBRM, there are no non-zero CPO values. Therefore none of the observations are surprising concerning the RCNBRM.

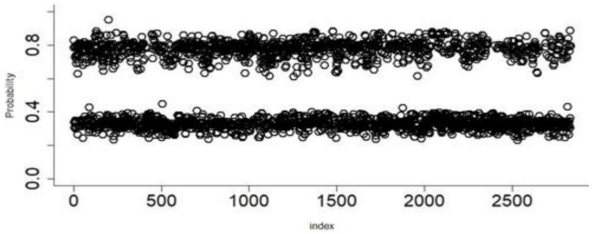

Figure 12.1: PIT values of RCNBRM

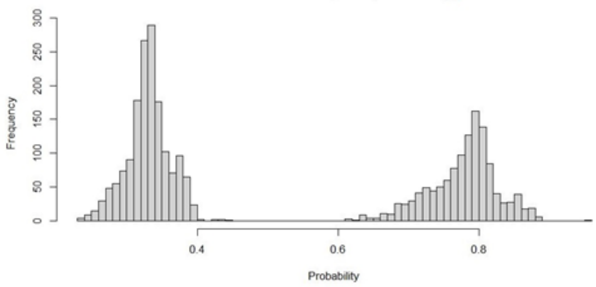

Figure 12.2: PIT values of RCNBRM

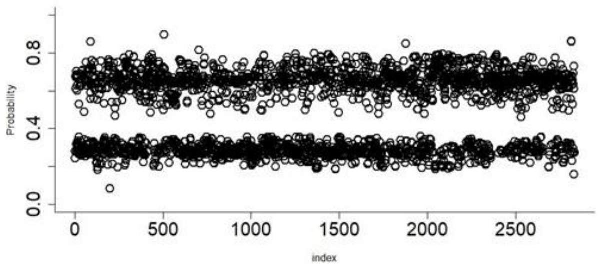

Figure 12.3: CPO values of RCNBRM

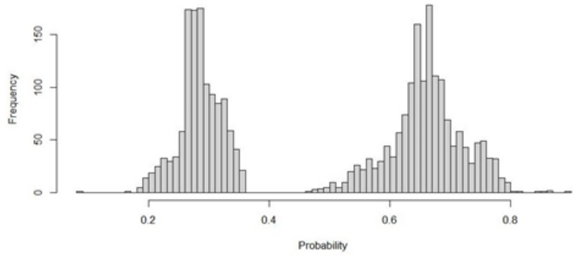

Figure 12.4: CPO values of RCNBRM

Figures 12.1 to 12.4 tells about the PIT and CPO values of RCNBRM respectively. In RCNBRM the PIT values do not indicate a lack of fit for the values do not appear to deviate from a uniform distribution

Table 6: Summary of values of marginal log-likelihood, DIC, and WAIC

	Model selection criteria		
Model	MLIK	DIC	WAIC
RCPRM	-2272.93	4467.14	4463.86
RCNBRM	-2274.29	4468.83	4465.08

Table 6 vividly explains the MLIK of RCPRM, -2272.93 is less than RCNBRM, -2274.29. The DIC of RCPRM, 4467.14 is less than RCNBRM, 4468.83, and the WAIC of RCPRM, 4463.86 is less than RCNBRM, 4465.08. Hence RCPRM is a better fit to the model right-censored NCSD. The RCPRM has lower DIC, WAIC, and MLIK values compared to RCNBRM. Hence it is more evidence that RCPRM is the correct model and better fit.

5. Discussion

This paper briefly describes the INLA algorithm to estimate marginal posterior mean or mode for parameters and hyperparameters for Bayes spatial to spatio-temporal models. The right-censored NCSD data set that fits

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RCPRM and RCNBRM is used to illustrate the INLA estimation results. The INLA produces great computational benefits rather than the other methods in solving problems that cover random and fixed effects to every specific region and time on its spatio-temporal analysis. In this study, the INLA additive model of fixed effects is computed. The RCPRM is the best-fit model compared to RCNBRM. The model right-censored NCSD can also be computed on other models. The right-censored number of caesarean section deliveries in Andhra Pradesh, India, 2019-2021 is modelled using Bayes spatial with INLA specification. The right-censored model NCSD can be compared with other regression models.

This study aimed to fit the model right-censored number of caesarean section deliveries using a count data regression model for the real-world data National Family Health Survey (NFHS-5), 2019-2021. The parameters of interest are estimated and compared to the right-censored number of caesarean-section deliveries (NCSD) among women of age 15-49, in the state of Andhra Pradesh, India. The RCPRM is found to be the best and concludes that during delivery, did you experience a breech presentation?, currently has heart disease, high blood pressure, prolonged labour, the child is twin, respondent's current age and highest educational level are important factors that determine the right-censored number of caesarean section deliveries.

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ORCID

The identifier is 0009-0004-4377-250X; iD and the link is https://orcid.org/0009-0004-4377-250X

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