



Modeling the Placement of Post Offices in a Certain Territory

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ABSTRACT:

Modeling and optimization criteria for the placement of post offices are proposed. Using the theory of queuing, there are general rules for determining the location of post offices in the district, allowing to minimize the average time spent by subscribers on delivering correspondence to the central postal facility. At the same time, the condition is met that the time spent by each subscriber remains approximately the same.

Introduction. In order to improve the quality and speed up the processes of processing postal items, as well as in accordance with the requirements of the postal rules and the Convention of the Universal Postal Union, the management of JSC "Uzbekistan Post" decided to automate the technological processes of processing postal items through the introduction of information technologies. One of the most urgent and at the same time the least studied theoretical problems of the organization of postal communication, in accordance with the decision, is the modeling of the placement of post offices (PO) in a certain area, which allows minimizing total costs, speeding up the processes of processing and sending mail.

Problem statement.

The theory of zoning of certain territories in the simplest case consists in dividing a given territory into some districts or service areas and evaluating the quality of the partitions according to the selected criterion.

Service areas, depending on the specific task being solved, can be large areas of territories (residential areas, industrial zones, the "core" or the city center), medium-sized areas (service areas of service centers, etc.), as well as local "areas" (for example, part of the territory, gravitating to a certain public organization).

The criteria for the quality of zoning may be to minimize the time of walking or public transport to the PO.

The assessment of the quality of various zoning options and the choice of the most acceptable option (optimal for this formulation of the problem) of software placement plays an essential role in building an effective scheme of walking, personal passenger, freight and public transport along the urban and rural road transport network. Therefore, it is of interest to create a certain theoretical basis that allows you to evaluate and optimize zoning. One of the goals of the zoning analysis is a preliminary assessment of the gain (or loss) obtained with various variants of discrete partitions of a given territory.

Solving the problem. With such a formulation of the problem, excessive complication of zoning algorithms will be a mathematical "excess" that does not bring any real economic gain, preferences, as with the optimization of a separate route [1], should be given to approximate asymptotic methods.

Only errors in the initial data, various practical and organizational factors, etc. do not allow for greater accuracy of calculations. The configuration of streets plays a particularly significant influence on the quality of zoning, especially in urban conditions. A real street network, as a rule, is far from the correct grid and in the conditions of Uzbekistan most often has the form of a random or, more precisely, a pseudo-random graph. Therefore, methods of modeling approaches and approaches to software based on optimization of flows over connected subgraphs on numerically labeled



graphs of a general form are of interest [2], With all the fundamental advantages of such an approach, it becomes difficult if the volume of the graph is large enough. Therefore, the methods of optimizing flows in graphs, often considered in the literature, are not very effective even with 100-1000 edges of graphs, i.e. for a large district or small regional city. This makes it necessary to decompose the flow optimization graph to the software, i.e. its division into smaller components (for example, neighborhoods using complex hierarchical systems of optimizing algorithms, or, much simpler, the use of approximate asymptotic formulas).

The first, most general, but not very accurate version of the asymmetric description of optimizing subscriber flows to the software, is the use of piecewise constant approximation of the density of distribution of subscriber flows over the territory of a city or other simulated territory. With all the inaccuracy, such a model is of fundamental importance in the theory of zoning, since it (and, most often, only it) makes it possible to find the optimal boundaries of service areas and the benefits of optimal zoning in general.

Let's consider and compare the three simplest zoning options:

1. Optimal zoning with minimal penalty function;
2. Zoning with the same capacity of districts, i.e. serving the same number of subscribers;
3. Zoning with the same areas of service areas.

In the asymptotic theory of data quantization, the most remarkable feature is the existence of a single extremum for all practically interesting objective functions of penalties, although exact expressions, with rare exceptions, have many local extremes. Using geometric terminology, we can say that the asymptotic approach corresponds to the replacement of a target polyhedron with many extremum vertices by a continuous smooth surface [3].

At the first stage of the discussion, we will take into account only the length of the tracks when moving a vehicle (vehicle) with subscribers or pedestrians inside each individual area. The results will generally be similar to quantization of a one-dimensional quantity, but there are also a number of features.

Let the service center for the n th service area (PO), which is a subdistrict of the G_p of the territory of this city, be located at the point of the cp of the territory, the density of the vehicle (or subscribers) is $w(x)$

vehicle/km² (or VEHICLE/Ha) at point x , and the area of the area is equal to H_n . Then the total length of paths (l_n) from PO to all points of this area (x) is represented by the expression

$$L_n = \sum_{x \in \Gamma_n} l(x, x_n) \quad (1)$$

where $l(x, x_n)$ is the distance between the point x and x_n in the urban metric.

Let us combine in the sum (1) the terms corresponding to the TC located approximately at the same distance $l(x, x_n)$ from the PO within a small area of ΔS_k . The number of such vehicles is obviously approximately equal to $w(x) \cdot \Delta S_k$, so that (1) takes the form:

$$L_n = \sum_{x_k \in \Gamma_n} w(x_k) l(x_k, x_n) \Delta S_k \quad (2)$$

The sum (2) exactly coincides with the integral of the step function $w(x) d(x, x_n)$ preserving a constant value on each section of ΔS_k . Following the principle of the asymptotic method, we replace this expression with the product $w(x) l(x, x_n)$ and get

$$L_n = \int_{\Gamma_n} w(x) l(x, x_n) dS \quad (3)$$

According to the mean theorem, this integral can be represented in the form:

$$L_n = w(x'_n) \int_{\Gamma_n} l(x, x_n) dS \quad (4)$$

where $w(x'_n)$ is the density at some point x'_n of the territory of the n th district.

Consider the integral included in (4). The parameter l has the dimension of length, i.e. the square root of the area S , and therefore the entire integral – dimension $S^{3/2}$ has the form:

$$l(x, x_n) dS = F_n H_n^{3/2}$$

where F_n is a dimensionless quantity, usually called a coefficient

forms or form factor.

Expression (4) takes the form:

$$L_n = F_n w(x'_n) H_n^{3/2} \quad (5)$$

For the case when the density is constant within the district, the district has a relatively simple shape. The directions of the paths, in this case, form a regular graph in the Euclidean or orthogonal (urban) metric, and the PO is located in the center of the district.

Formula (5) is well known from the literature on zoning [4]. It is also valid in a much more general



situation, i.e. for any density, shape of the area, direction of routes, location of software, etc. Obviously, all these factors have an impact on the value of F_n .

However, as is well known from design experience, this dependence is weak for real conditions, so for the purposes of preliminary assessment it is quite acceptable to replace it with an average value of F .

From (5) it is possible to find the total distance of movement of subscribers containing N BY:

$$L = \sum_{n=1}^N L_n = \sum_{n=1}^N F_n w(x'_n) H_n^{3/2}$$

(6)

We introduce a piecewise constant function $\varphi(x)$ equal to a fixed value on the territory of each (n th) district. $F_n w(x'_n) H_n^{3/2} = \varphi(x_n)$. Then (6) is exactly equal to the integral over the territory of the city of:

$$L = \sum_{n=1}^N \varphi(x_n) H_n = \int \varphi(x) dS$$

If we consider H_n and G_p as functions of the coordinate of the location of the PO, $x = x_n$ and approximately replace $w(x'_n)$ with $w(x_n) = w(x)$, it follows from (7)

$$L = \int F(x) w(x) (H(x))^{1/2} dS$$

Let us explain the meaning of this asymptotic formula. With discrete calculations, areas with software are distinguished by their numbers n , and numbering is done in an arbitrary order and does not directly reflect the position of software in the territory. Therefore, with a continuous approach, it is advisable to apply "natural" designations, characterizing the parameters of the districts with coordinates corresponding to, $x = x_p$, $N_p = H(x_p)$, $F_n = F(x_p)$, etc.

The latter formula implies the replacement of the step function $\varphi(x)$ by a continuous, smooth integrand function. The approach has an asymmetric character, since the accuracy of such a substitution increases with an increase in the number of N , as well as for smooth $W(x)$.

To assess the placement of software on the territory of the city, it is advisable to introduce the concept of station density $V(x)$, which characterizes the number of software located on a plot of territory with an unambiguous area and center X :

$$V(x) = 1/H(x) [10^3/\text{km}^2]$$

(8)

The expression for the total path length taking into account (8) takes the form:

$$L = \int F(x) W(x) (V(x))^{-1/2} dS$$

(9)

Γ

Next, we will estimate the "costs" (time, fuel, etc.) when moving along the tracks. The "cost" of each subscriber path is assumed to be a linear function of its length $l(x, x_n)$:

$$P(x, x_n) = P_0 + P_1 l(x, x_n),$$

(10)

where P_0 is the initial cost. Independent of the path length;

P_1 – costs for 1 km of the way.

Summing up (10) for all possible paths, we get:

$$P = \sum_{x \in \Gamma} P(x, x_n) = P_0 M + P_1 L$$

(11)

Here M is the total number of paths, i.e. the number of subscribers (capacity) of the PO.

Next, we calculate the cost of the distance of connecting paths between different zones. The cost of each of these paths having length $l(x, x_n)$ and passing load $y(x_1, x_2)$ [TC/hour, ab./hour] is approximated by the function:

$$q(x_1, x_2) = q_1 l(x_1, x_2) + q_2 l(x_1, x_2) y(x_1, x_2) + q_3 y(x_1, x_2) + q_4 \quad (12)$$

If the software is located at points x_1, x_2 and has capacities M_1 and M_2 , their mutual load, as usual, is estimated by the formula:

$$y(x_1, x_2) \approx y_0 M_1 M_2 / M = y_0 \int w(x') ds' / \int w(x'') ds'' / M \approx$$

$$\frac{\int_{\Gamma_1} w(x_1) H(x_1) w(x_2) H(x_2) / M}{\int_{\Gamma_1} w(x_1) H(x_1) v(x_1) v(x_2) H(x_2) / M} = y_0 w(x_1) H(x_1) v(x_1) v(x_2) H(x_2) / M \quad (13)$$

To find the total cost of connecting paths, we take into account that in each section of the territory with an area of $\Delta S(x)$, $\Delta S(x)$ is located $H(x) = V(x) \Delta S(x)$. Therefore, when connecting all the software located on the site of ΔS_1 and ΔS_2 , according to the "each with each" scheme, $V(x_1)$ is required $\Delta S(x_1) V(x_2) \Delta S(x_2)$ of lines (bundles). Their cost is

$$q(x_1, x_2) v(x_1) v(x_2)$$

$\Delta S_1 \Delta S_2$,

so the total cost of connecting lines is:

$$Q = \iint q(x_1, x_2) v(x_1) v(x_2) ds_1 ds_2 \quad (14)$$

$\Gamma \Gamma$

When substituting expressions (12) and (13) in (14), we get:

$$Q = q_1 \iint l(x_1, x_2) v(x_1) v(x_2) ds_1 ds_2 + (q_2 y_0 / M) \iint l(x_1, x_2) w(x_1) w(x_2) ds_1 ds_2 +$$

$\Gamma \Gamma$

$\Gamma \Gamma$



$$\int v(x) ds)^2 \quad (15)$$

Obviously, the integral $M = \int w(x) ds$ is equal to the total capacity of all RO

$N = \int v(x) ds$ the number of software; the other two integrals included in (15)

represent the total lengths of paths:

$$L' = \int \int l(x_1 x_2) v(x_1) v(x_2) ds_1 ds_2$$

$$L'' = \int \int l(x_1 x_2) w(x_1) w(x_2) ds_1 ds_2$$

$$L'' = \int \int l(x_1 x_2) w(x_1) w(x_2) ds_1 ds_2$$

The value of L_{11} is equal to the total length of the connecting paths, and L_1 is the sum of the distances between any points within the area.

Thus, the total costs of connecting lines, as follows from (15), are equal to:

$$Q = q_1 L' + (q_2 y_0 / M) L'' + q_3 y_0 M + q_4 N^2 \quad (17)$$

Finally, it is advisable to approximate the costs of building PO and PO with an asymptotic approach by a simple linear function of their capacity M_n :

$$R_n = r_0 + r_1 M_n \quad (18)$$

Then the total cost of all software is represented by the expression:

$$R = \sum_{n=1}^N R_n = r_0 N + r_1 M \quad (19)$$

where, as above, N is the amount of PO, M is the capacity of PO.

From formulas (11), (17), (19) this implies an asymptotic estimate of the part of the total costs for the construction or development of PO and PO, which depends on the zoning, i.e. on the number and location of PO:

$$v = P + Q + R = P_0 M + q_3 y_0 M + r_1 M + q_4 N^2 + r_1 N + P_1 L + q_1 L' + q_2 y_0 L'' / M \quad (20)$$

Formula (20) is written for the case when all the equipment used in the districts are of the same type. However, its generalization to more complex tasks does not cause fundamental difficulties.

Despite the approximate nature of the asymptotic approach, experience shows that the calculation results have quite acceptable accuracy, especially if several

variants calculated using the same method are compared (errors for all variants are largely compensated). This is confirmed by the fact that, as can be seen from (20), this formula generalizes and clarifies methods widely used in domestic and foreign studies of design problems.

The accuracy and validity of the recommendations obtained on the basis of the asymptotic approach largely depends on the validity of the parameters taken in the calculation of the values of the cost P_i, q_i, r_i .

To calculate the cost values (20), it is also required to specify the shape of the territory occupied by the city (the area of integration D), the distribution of vehicles or subscribers in this territory (their surface density $w(x)$), averaged parameters characterizing the difference between the lengths of paths from the shortest Euclidean distances (the function of the form factor $F(x)$). The ability to trace the dependence of costs on these factors shows the usefulness of the asymptotic approach.

Summing up the above, we can assume that the asymptotic calculation of the PO and PO options is very useful as the first, preliminary stage of choosing the characteristics of the service network as a whole. However, the results of such a calculation are not sufficient for the purposes of practical design. Indeed, it follows from formula (20) and the parameters L, L_1, L_{11} included in it that they do not allow directly determining the optimal or expedient coordinates of the PO and the boundaries of the station areas. This part of the network synthesis should be performed on the basis of more detailed calculations described in the following paragraphs.

At the same time, the asymptotic approach is suitable for solving a fairly wide range of problems, as examples we will indicate:

1. Determination of the optimal quantity and density of the distribution, which are connected by an obvious formula:

$$N = \int v(x) ds \quad (21)$$

2. Comparison of different zoning methods for given amounts of the same type of software.

3. Comparison of the costs of building or developing an urban transport network using equipment of various systems.

4. Choosing the optimal forecast for the development



of urban transport networks.

Most of these problems can be solved in an analytical form using conventional methods of differential and variational calculus.

However, these calculations, as a rule, require setting a significant number of cost parameters, which are difficult to obtain. Therefore, we will limit ourselves to just one special case directly related to the topic of the work – a comparison of zoning options by the "cost" of paths.

Suppose that these urban transport routes (with the given $W(x)$, $F(x)$ and D) contain a fixed number of N of the same type of districts. Then the various variants of a fully connected network are completely determined by the placement of the software, characterized in the asymptotic model by the "station" density $V(x)$; the length of the intra-district and connecting paths (L and L^1) are expressed in terms of this density. Therefore, when comparing options, we can limit ourselves only to those terms (20) that depend on $V(x)$:

$$v_0 = P_1 L + q_1 L^1 \quad (22)$$

Discarding the constant part of the costs exaggerates the differences in options, which corresponds to the assessment of the gain from below.

For the methods of constructing urban transport routes (GTP) listed at the beginning of the paragraph, we obtain the following relations:

1. When constructing a GP with constant capacities of software, M_p , all subscribers of the network are divided into N groups with the same numbers of $M_p = M / N$. Each group is served by one of the software. The station density is determined from the formula:

$$M_n = \int W(x) ds \approx W(x_n) H(x_n) = W(x_n) / V(x_n) \quad (23)$$

Hence follows:

$$V(x) = W(x) / M_n = N W(x) / M$$

$$L = L_c = \sqrt{M/N} \int F(x) W(x)^{-1/2} ds = 1/\sqrt{N} \left(\int W(x) ds \right)^{1/2} \int F(x) W(x)^{-1/2} ds \quad (24)$$

$$L^1 = L_c^1 = (N/M)^2 \int l(x_1, x_2) W(x_1) W(x_2) ds_1 ds_2 =$$

$$N^2 \left(\int W(x) ds \right)^{-2} \int l(x_1, x_2) W(x_1) W(x_2) ds_1 ds_2$$

$$W(x_2) ds_1 ds_2$$

$$\Gamma \quad \Gamma$$

$$L^1 = (N/M)^2 L^1$$

(25)

Thus, the total length of the connecting paths between the PO in this case is proportional to the sum of the distances between the sensing points.

2. When constructing a GTR with permanent areas of station districts, the entire area of the city S is divided into N equal - sized areas with areas

$$H_n = S/N$$

Accordingly, the station density is constant: $V(x) = 1/N(x) = N/S$.

Formulas (9) and (16) take the form:

$$L = L_m = \sqrt{S/N} \int F(x) W(x)^{-1/2} ds = 1/\sqrt{N} \left(\int ds \right)^{1/2} \int F(x) W(x) ds$$

$$\Gamma \quad \Gamma$$

$$L^1 = L_m^1 = (N/S)^2 \int \int l(x_1, x_2) ds_1 ds_2 = N^2 \left(\int ds \right)^{-2} \int l(x_1, x_2) ds_1 ds_2$$

$$\Gamma \quad \Gamma$$

3. Finally, when zoning a GTR with a minimum cost inside the district tracks, it is necessary to determine the function $V(x)$, at which the value of the functional $P_1 L$ of (22) takes the smallest value possible, provided that the number of stations N , determined by the integral (21), is fixed. This problem is equivalent to finding the conditional minimum of the total length of paths expressed by the functional (8).

To solve it, we apply the method of indefinite multipliers, the Lagrange functional [5]. The unconditional extremum of which coincides with the desired conditional extremum:

$$\Phi = L + \lambda N = \int F(x) W(x) (V(x))^{-1/2} ds + \lambda \int V(x) ds = \int [F(x) W(x) (V(x))^{-1/2} + \lambda V(x)] ds$$

(26)

Obviously, integral (26) takes an extreme value if, for any fixed value $x_n = x_n$, its subintegral function also takes such a value. Thus, it is possible to differentiate this function by the symbol $M = M(x)$, where x is considered as a fixed parameter. Let's put:

$$\Psi = F W V^{-1/2} + \lambda V,$$

Then the extremum conditions take the form:

$$d\Psi/dV = -\frac{1}{2} F W V^{-1/2} + \lambda = 0 \quad (27)$$



$$d^2 \Psi / dV^2 = \frac{3}{4} FWV^{-5/2} > 0$$

The last inequality shows that the found extremum is the minimum. Accordingly, (27) really determines the stationary density:

$$V(x) = (FW/2\lambda)^{2/3} \quad (28)$$

The parameter λ is determined by substituting (28) into (26)

$$N = (2\lambda)^{-2/3} \int_{\Gamma} F(W)^{2/3} ds \quad (29)$$

$$\lambda = 1/2 \left(\frac{1}{N} \int_{\Gamma} F(x)W(x)^{2/3} ds \right)^{3/2}$$

Hence the final formula for station density follows:

$$V(x) = N F(x)W(x)^{2/3} / \int_{\Gamma} (F(r)W(z))^{2/3} ds \quad (30)$$

When substituting (29), the first formula (27) can be written in the form:

$$F(x_n)W(x_n)(H(x_n))^{3/2} = 2\lambda = \left(\frac{1}{N} \int_{\Gamma} (F(x)W(x))^{2/3} ds \right)^{3/2} = \text{const} \quad (31)$$

Comparison of (31) and (5) shows that with optimal zoning, the total lengths of the routes of all service areas should have the same values. It follows from the above conclusion, that in the asymptotic approximation this criterion is also valid in the general case.

From (31), (9) and (23) we find:

$$L = L_0 = (1/\sqrt{N}) \left(\int_{\Gamma} (F(x)W(x))^{2/3} ds \right)^{3/2};$$

$$L^1 = L_0^1 = N^2 \left(\int_{\Gamma} (F(x)W(x))^{2/3} ds \right)^{-2} \int_{\Gamma} l(x_1, x_2) (F(x_1)W(x_1))^{2/3} (F(x_2)W(x_2))^{2/3} ds_1 ds_2$$

Similarly, it is possible to set a more general task of minimizing the total cost of district and inter-district connecting paths. However, finding the extremum of such a functional leads to a nonlinear integral equation, the solution of which is rather cumbersome.

Let's compare the indicators of the three zoning options based on the formulas (24), (25), (31), and also (26).

First of all, consider the same type of expressions for L_c , L_n , L_0 .

Obviously, the ratio of these values to the number of paths between PO N^2 makes sense of the average lengths of paths:

$$l_c = L_c / N^2; \quad l_n = L_n / N^2; \quad l_0 = L_0 / N^2; \quad (32)$$

From the above formulas it can be seen that the placement of software on the territory of the city is uniform when zoning with permanent areas and uneven in other cases, when the software is concentrated in areas with increased vehicle (subscribers). The degree of this concentration is different for optimal zoning and for zoning with a constant capacity. Indeed, from the above expressions $V(x)$ it can be seen that with an increase in the density of $W(x)$, for example, by 10 times, the area of N_p areas decreases in the first case by $10^{2/3} \approx 4.64$ times, and in the second case - by 10 times. Accordingly, in the high-density zone, where the majority of PO is concentrated, the connecting paths in the first case will be reduced by approximately $(10^{2/3})^{1/2} = 10^{1/3} \approx 2.15$ times, and in the second case - by $10^{1/2} \approx 3.16$ times.

From these considerations, it can be seen that for any city where the density of $W(x)$ has a small value in the center and decreases to the periphery, the smallest total length of connecting paths is provided by zoning with constant capacities of PO, in the longest length of these paths – zoning with constant areas of station areas.

The optimal (along the length of the paths) zoning occupies an average position. The conclusion can be easily confirmed by calculating the integral expressions L_c^1 , L_p^1 , L_0^1 for specific G and $W(x)$, the difference in options depends on the specific characteristics of cities. Let's start comparing the lengths of subscriber paths with L_c and L_a , defined by formulas (24) and (25). When replacing $F(x)$ with the average value (see above), their ratio is equal to

$$\alpha = L_c / L_H = \int_{\Gamma} W(x)^{1/2} ds / \left(\int_{\Gamma} W(x) ds \right)^{1/2} \quad (33)$$

This expression can be evaluated using the Bunyakovsky–Schwartz integral inequality [6]:

$$\int_{\Gamma} f(x) ds \leq \left(\int_{\Gamma} ds \right)^{1/2} \left(\int_{\Gamma} f^2(x) ds \right)^{1/2}$$

Assuming in this inequality that are valid for all functions for which there are integrals $f(x) = (W(x))^{1/2}$ we get $\alpha \leq 1$, i.e. the length of the path when zoning with constant software capacities is always no longer than when zoning with constant areas of districts. In this case, equality is achieved only when the density is constant throughout the city, $W(x) = \text{const}$, when both zoning options coincide.

To compare options with a minimum length of paths and with a constant RO capacity, it is advisable to make an expression of the "gain" of optimal zoning



(also for a certain shape coefficient):

$$g = (L_c - L_0) / L_0 = \int_{\Gamma} (W(x))^{1/2} ds \int_{\Gamma} (W(x) ds)^{1/2} / (\int_{\Gamma} (W(x))^{2/3} ds)^{3/2} \quad (34)$$

To study this expression to the maximum, we should find the function $W(x)$ corresponding to the largest value of the functional (34). However, the use of standard methods of calculus of variations shows that this problem has no solution in the class of smooth functions – the gain increases indefinitely with the approximation of $W(x)$ to the Dirac delta function [7]. To get around this difficulty, we will determine the maximum (34) with a fixed ratio of the largest and smallest values on the territory of the city:

$$g = W_{max} / W_{min} = W_M / W_m \quad (35)$$

It is not difficult to verify that in this case the limit of the extremum is reached if $W(x)$ is equal in each or W_v or W_m . Thus, the problem under consideration leads to a "two-level" model of the city.

Suppose that $W(x) = W_m$ on the part of the city with area S and $W(x) = W_v$ on the part with area $S_v = S - S_m$. then (35) takes the form:

$$g = ((\sqrt{W_M} S_M + \sqrt{W_m} S_m) \sqrt{W_M} S_M + W_M S_M) / (W_M^{2/3} S_M + W_m^{2/3} S_m)^{3/2} \quad (36)$$

Let's introduce the notation:

$$S_M / S_m = \beta, \quad x = q^{1/6}$$

Formula (36) takes the form:

$$g = \{[(1 + \beta x^3)(1 + \beta x^6)^{-1/2}] / (1 + \beta x^4)^{3/2}\} - 1 \quad (37)$$

Let's choose the area ratio so that g is maximal. To do this, calculate the derivative:

$$\partial g / \partial \beta = [(g x^3 (x-1)) / 2(1 + \beta x^4)(1 + \beta x^6)(1 + \beta x^3)] [(x-1)(x+2) - x^4(2x^2 - x + 1) \beta]$$

Equating this expression to zero, we find:

$$\beta = \beta_0 = [(x-1)(x+2)] / x^4(2x^2 - x + 1)$$

The derivative $\partial g / \partial \beta$ at a stationary point will change the sign from plus to minus for all values of x , so that the stationary point is the maximum. Substituting $\beta = \beta_0$ in (37), we obtain the calculated formula:

$$g_{max} = [2(x^3 + x - 1) / x(3x^2 - 1)] \sqrt{(x^4 + x^3 - x + 1) / (3x^2 - 1)} \quad (38)$$

The results of calculations of the largest gain are shown in Table 1.

The greatest gain is g_{max} and g_{kr} Table 1

g	3.333	10	33,33	100	333,3
β	0,1100	0,0910	0,0515	0,0274	0,0129
g_{max}	0,0103	0,049	0,1250	0,2280	0,3790
g_{kr}	0,0049	0,0160	0,0320	0,0470	0,0600

The results obtained are approximated with satisfactory accuracy by a simple formula:

$$g_{max} = 0,385(g^{1/6} - 1) + 0,405 (g^{-1/6} - 1) \quad (39)$$

It can be seen from the table that the gain values for other functions $W(x)$, although with a given uneven distribution $g(32)$, the gain turns out to be somewhat less. For example, for the territory of a city in the shape of a circle and the density $W(r) = e^{-kr}$, $r^2 = x^2 + y^2$, you can get the formula:

$$g = g_{kp} = (4\sqrt{6} / 9)[(q^{1/2} - 1)(q - 1)^{1/2} / (q^{2/3} - 1)^{3/2}] \quad (40)$$

The results of the calculation using this formula are also shown in Table 1.

The obtained very insignificant values of the gain should be additionally adjusted taking into account the fact that for connecting paths, it is more advantageous

to build a network with constant RO capacities. The losses in the length of these paths for the considered cases have the same order of magnitude as the gains indicated in Table.3.1.

Conclusion. The issues of the theory of zoning and modeling "in general" on the placement of post offices in cities and other territories are considered. The obtained results provide a theoretical basis for further research of this complex problem.

Calculations show that due to the low sensitivity of the optimal zoning gain from small errors in determining the position of service centers and the boundaries of service areas, it is recommended to use a relatively simple asymptotic approach to solve the problem.

As a result of the calculation, it was found that theoretically optimal is zoning with the same total or



average length of all internal routes of subscribers for all zones (generalization of the criterion of I.P.Zhdanov).

Zoning with the same number of subscribers in all zones is only a few percent units inferior to optimal zoning. A more detailed analysis showed that it is rational to use such zoning in most practical cases.

The main conclusion from the calculation is that zoning with constant capacities of service areas is a completely satisfactory approximation to zoning with the maximum total cost for passage along intra- and inter-district routes. This option minimizes the costs of choosing urban transport routes, therefore it can be considered as a practically optimal approach to their design according to the criterion of minimum total costs for development or construction. The reserve of savings with this method of zoning is the correct choice of the standard capacity, and therefore the number of districts, improving the structure of the ways of approach to post offices, but not the choice of the optimal capacity of each individual district.

At the same time, it should be noted that the above calculation of the PO and software options is very useful as the first, preliminary stage of choosing the characteristics of the service network as a whole. However, the results of such a calculation are not sufficient for practical design purposes, it is necessary to obtain equations for optimal zoning of PO.

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