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On Type-I And Type-II Istratescu α-Almost Contractions In B-Metric Spaces

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ABSTRACT:

In this paper, we extend the concepts of Type-I and Type-II Istratescu α-almost contractions to b-metric spaces. We explore the existence of fixed points for both Type-I and Type-II mappings, and the obtained results are exemplified through various examples to provide a comprehensive understanding.

Introduction

Istratescu [1], [2] proved an interesting fixed point theorems in 1982 and 1983.

Theorem 1.1. [1] Suppose (Θ, d) is a complete metric space, and $\xi: \Theta \to \Theta$ is a self-map. Suppose that there exist $a_1, a_2 \in (0,1)$ such that $a_1 + a_2 < 1$ and
$d(\xi^2\theta, \xi^2\psi) \le a_1 d(\xi\theta, \xi\psi) + a_2 d(\theta, \psi)$ for all $\theta, \psi \in \Theta$. Then ξ has at most one fixed point.
Berinde [3] presented an intriguing extension of the contraction mapping under the heading of almost contraction.

Definition 1.1. A self-mapping ξ on a metric space (Θ, d) is called almost contraction if there exists a constant $\kappa \in (0,1)$ and $\varrho \ge 0$ such that
(1.2) $\quad d(\xi\theta, \xi\psi) \le \kappa d(\theta, \psi) + \varrho d(\psi, \xi\theta)$ for all $\theta, \psi \in \Theta$.
The concept of a metric space has been expanded in diverse ways, enabling the extension of the previously mentioned contraction principle to these new contexts.

Definition 1.2. Let Θ be a nonempty set. Then, a mapping $d: \Theta \times \Theta \to [0, +\infty)$ is called a b-metric space if there exist a number $s \ge 1$ such that for all $\theta, \psi, \omega \in \Theta$,
(b_1) $d(\theta, \psi) = 0$ if and only if $\theta = \psi$.
(b_2) $d(\theta, \psi) = d(\psi, \theta)$ for all $\theta, \psi \in \Theta$.
(b_3) $d(\theta, \psi) \le s[d(\theta, \omega) + d(\omega, \psi)]$
Then the pair (θ, d) is called a b-metric space with index s.
Clearly, every metric space is a b-metric space with $s = 1$. Unlike the standard metric, due to the modified triangle inequality, b-metric is not always continuous. (See, eg., [4]).
We need the following lemma in a b-metric space (See, eg., [[5]-[10]] and references therein) for further development.

Lemma 1.1. Every sequence $\{\theta_n\}$ with elements from a b-metric space (Θ, d, s) satisfies for every $n \in N$ the inequality $d(\theta_0, \theta_r) \le s^n \sum_{i=0}^{r-1} d(\theta_i, \theta_{i+1})$ where $r \in \{1,2,3, \dots, 2^n - 1, 2^n\}$.
One way to describe the Cauchy criteria in a b-metric space is as follows. (See, eg., [10]).

Lemma 1.2. Let $\{\theta_n\}$ be a sequence of elements in a b-metric space (Θ, d, s). Then $\{\theta_n\}$ is a cauchy sequence if there exist $l \in [0, 1)$ such that $d(\theta_n, \theta_{n+1}) \le l d(\theta_n, \theta_{n-1})$ for every $n \in N$.

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Definition 1.3. Let Θ be a nonempty set, $\alpha: \Theta \times \Theta \to [0, \infty)$ and $\xi: \Theta \to \Theta$ be mapping such that

(1.5) $\alpha(\theta, \xi\theta) \geq 1 \Rightarrow \alpha(\xi\theta, \xi^2\theta) \geq 1$, for all $\theta \in \Theta$.

Following that, ξ is referred to as a ”α-orbital admissible mapping” [11].

In the context of a b-metric space, this paper explores two new categories of generalized contractions. We establish the conditions for the existence of fixed points for such mappings and delve into practical implications, supported by illustrative examples.

Main Results

Let us begin by introducing the Type-I IStratescu α -almost Contraction.

Definition 2.1. Let (Θ, d) be a complete b-metric space with index $s \geq 1$ and a function $\alpha: \Theta \times \Theta \to [0, \infty)$. A mapping $\xi: \Theta \to \Theta$ is called Type-I IStratescu α-almost Contraction if there exist $\kappa \in [0,1)$ and $\Lambda \geq 0$ such that for any $\theta, \psi \in \Theta$

(2.1) $\alpha(\theta, \psi)d(\xi^2\theta, \xi^2\psi) \leq \kappa. \Xi(\theta, \psi) + \Lambda. \eta(\theta, \psi)$

where

(2.2) $\Xi(\theta, \psi) = d(\xi\theta, \xi\psi) + |d(\xi\theta, \xi^2\theta) - d(\xi\psi, \xi^2\psi)|$

and

(2.3) $\eta(\theta, \psi) = \min \{d(\theta, \xi\theta), d(\psi, \xi\psi)\}$

Now we prove the following result.

Theorem 2.1. Let (Θ, d) be a complete b-metric space with index $s \geqslant 1$ and $\xi: \Theta \to \Theta$ on Type-I IStratescu α-almost Contraction such that ξ is continuous, if ξ is α-almost admissible and there exist $\theta_0 \in \Theta$ such that $\alpha(\theta_0, \xi\theta_0) \geq 1$. Then ξ has a fixed point. If further $\Lambda + \kappa < 1$, then fixed point is unique.

Proof: Let $\theta_0 \in \Theta$ be the given point with the property that $\alpha(\theta_0, \xi\theta_0) \geq 1$.

Because the mapping has the α-orbital admissible property, we have that $\alpha(\xi\theta_0, \xi^2\theta_0) \geq 1$, and continuing this process we get,

(2.4) $\alpha(\xi^n\theta_0, \xi^{n-1}\theta_0) \geq 1$, for $n \in N$

Replace θ by θ_0 and ψ by $\xi\theta_0$ in (2.1), we have

$d(\xi^2\theta_0, \xi^3\theta_0) \leq \alpha(\theta_0, \xi\theta_0)d(\xi^2\theta_0, \xi^3\theta_0) \leq \kappa. \Xi(\theta_0, \xi\theta_0) + \Lambda. \eta(\theta_0, \xi\theta_0)$

(2.5) $d(\xi^2\theta_0, \xi^3\theta_0) \leq \kappa(d(\xi\theta_0, \xi^2\theta_0) + |d(\xi\theta_0, \xi^2\theta_0) - d(\xi^2\theta_0, \xi^3\theta_0)|)$
$+\Lambda. min\{d(\theta_0, \xi\theta_0), d(\xi\theta_0, \xi^2\theta_0)\}$

Case -1: $d(\xi\theta_0, \xi^2\theta_0) \leq d(\xi^2\theta_0, \xi^3\theta_0)$

Therefore $d(\xi^2\theta_0, \xi^3\theta_0) \leq \kappa d(\xi^2\theta_0, \xi^3\theta_0) + \Lambda\, d(\xi\theta_0, \xi^2\theta_0)$

Therefore

(2.6) $d(\xi^2\theta_0, \xi^3\theta_0) \leq \frac{\Lambda}{1-\kappa} d(\xi\theta_0, \xi^2\theta_0)$ $\left(Since\ \frac{\Lambda}{1-\kappa} < 1\ and\ \Lambda + \kappa < 1\right)$.

(2.7) $d(\xi^2\theta_0, \xi^3\theta_0) \leq \frac{\Lambda}{1-\kappa} d(\xi^2\theta_0, \xi^3\theta_0)$

Therefore $d(\xi^2\theta_0, \xi^3\theta_0) = 0$

Therefore $d(\xi\theta_0, \xi^2\theta_0) = 0$

Therefore θ_0 is a fixed point of ξ.

Case-2: $d(\xi\theta_0, \xi^2\theta_0) > d(\xi^2\theta_0, \xi^3\theta_0)$

Therefore

$d(\xi^2\theta_0, \xi^3\theta_0) \leq 2\kappa d(\xi\theta_0, \xi^2\theta_0) - \kappa d(\xi^2\theta_0, \xi^3\theta_0) + \Lambda d(\xi\theta_0, \xi^2\theta_0)$

$d(\xi^2\theta_0, \xi^3\theta_0) \leq \frac{2\kappa + \Lambda}{1 + \kappa} d(\xi\theta_0, \xi^2\theta_0)$

(2.8) $= \mu d(\xi\theta_0, \xi^2\theta_0)$ $\left(Since\ \mu = \frac{2\kappa+\Lambda}{1+\kappa} < 1\right)$

For $\theta = \xi\theta_0, \psi = \xi^2\theta_0$ in (2.1)

(2.9) $\alpha(\xi\theta_0, \xi^2\theta_0)d(\xi^3\theta_0, \xi^4\theta_0) \leq \kappa(d(\xi^2\theta_0, \xi^3\theta_0) + |d(\xi^2\theta_0, \xi^3\theta_0) - d(\xi^3\theta_0, \xi^4\theta_0)|)$
$+\Lambda min\{d(\xi\theta_0, \xi^2\theta_0), d(\xi^2\theta_0, \xi^3\theta_0)\}$

In general, we can show that

(2.10) $d(\xi^n\theta_0, \xi^{n+1}\theta_0) \leq \left(\frac{2\kappa+\Lambda}{1+\kappa}\right)^n d\left(\xi\theta_0, \xi^2\theta_0\right) \leq \left(\frac{2\kappa+\Lambda}{1+\kappa}\right)^n d\left(\theta_0, \xi\theta_0\right) \to as\ n \to \infty.$

However, taking into account the sequence $\{\theta_n\}_{n\in N}$ defined as follows

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$$\theta_1 = \xi\theta_0, \theta_2 = \xi^2\theta_0, \theta_n = \xi^n\theta_0 \ where\ \theta_0 \in \Theta.$$

From Equation (2.9), we have

$d(\theta_n, \theta_{n+1}) \le c d(\theta_{n-1}, \theta_n)$ $for\ n \in N, where\ c = \frac{2\kappa+\Lambda}{1+\kappa}$

As a result, Lemma 1.2 tells us that in the complete b-metric space Θ, $(\theta_n)_{n\in N}$ forms a Cauchy sequence.
As a result, it is convergent, and there is $u \in \Theta$ and $\ni$

(2.11) $\lim_{n\to\infty} d(\theta_n, u) = 0$

As a result of the mapping ξ being continuous, it is evident that
$\lim_{n\to\infty} d(\theta_n, \xi u) = \lim_{n\to\infty} d(\xi\theta_{n-1}, \xi u) = 0$ and since u is a fixed point of ξ i.e., $\xi u = u$.

Uniqueness

Suppose $\Lambda + \kappa < 1$. Let θ, ψ be two fixed points of ξ. Then

$$\alpha(\theta,\psi)d(\xi^2\theta, \xi^2\psi) \le \kappa(d(\xi\theta, \xi\psi) + |d(\xi\theta, \xi^2\theta) - d(\xi\psi, \xi\psi)|) + \Lambda. min\{d(\theta, \xi\theta), d(\psi, \xi\psi)\}$$

$\therefore d(\theta,\psi) \le \kappa(d(\theta,\psi) + |d(\theta,\theta) - d(\psi,\psi)|) + \Lambda. min\{d(\theta,\theta), d(\psi,\psi)\}$

$\therefore d(\theta,\psi) \le \kappa(d(\theta,\psi)) \Rightarrow (1 - \kappa - \Lambda)d(\theta, \psi \le 0).$

$$\Rightarrow d(\theta,\psi) = 0 \Rightarrow \theta = \psi.$$

$\therefore$ The fixed point is unique.

Note: Theorem-2.1 is proved in Istratescu [1] with $\eta(\theta,\psi)$ replaced by $\eta(\theta,\psi) = min\{d(\theta,\xi\theta), d(\psi,\xi\psi)\}$.

Definition 2.2. Let $\alpha: \Theta \times \Theta \to [0,\infty)$ satisfy

(i) $\alpha(\theta,\psi) = \alpha(\psi,\theta)$

(ii) $\alpha(\theta,\psi) \ge 1, \alpha(\psi,\omega) \ge 1 \Rightarrow \alpha(\theta,\omega) \ge 1.$

Then α is said to be a triangular function on Θ.

Note: Suppose α is triangular and ξ is α-orbital then ξ^2 is α-orbital.
Since $\alpha(\theta,\xi\theta) \geqslant 1 \Rightarrow \alpha(\xi\theta,\xi^2\theta) \geqslant 1 \Rightarrow \alpha(\xi^2\theta, \xi^3\theta) \geqslant 1 \Rightarrow \alpha(\xi^3\theta, \xi^4\theta) \geqslant 1$.
Now ξ^2 is α-orbital means $\alpha(\theta,\xi^2\theta) \geqslant 1 \Rightarrow \alpha(\xi^2\theta,\xi^4\theta) \geqslant 1$.

Lemma 2.1. If α is triangular and ξ is α-orbital admissible, then ξ^2 is α-orbital admissible.

Definition 2.3. Let (Θ, d) be a complete b-metric space with index $s \geqslant 1$ and a function $\alpha : \Theta \times \Theta \to [0,\infty)$. A mapping $\xi : \Theta \to \Theta$ is called Type-II IStratescu α-almost Contraction if there exist $\kappa \in [0,1)$ and $\Lambda \geqslant 0$ such that for any $\theta,\psi \in \Theta$

(2.12) $\alpha(\theta,\psi)d(\xi^2\theta,\xi^2\psi) \le \kappa\Xi^*(\theta,\psi) + \Lambda\eta(\theta,\psi)$

where

(2.13) $\Xi^*(\theta,\psi) = |d(\theta,\xi\theta) - d(\xi\psi,\xi^2\psi)| + d(\theta,\psi) + |d(\psi,\xi\psi) - d(\xi\theta,\xi^2\theta)|$

and

(2.14) $\eta(\theta,\psi) = min\{d(\theta,\xi\theta), d(\psi,\xi\psi)\}$

Theorem 2.2. Let (Θ, d) be a complete b-metric space with index $s \geqslant 1$ and $\xi: \Theta \to \Theta$ is an Type-II IStratescu α-almost Contraction such that ξ^2 is continuous and suppose that α is triangular, ξ is α-almost admissible and there exist $\theta_0 \in \Theta$ such that $\alpha(\theta_0, \xi\theta_0) \geqslant 1$, consequently ξ^2 has a fixed point.

Proof. Let $\psi = \xi^2\theta$, in (2.12), then

(2.15) $\alpha(\theta,\xi^2\theta)d(\xi^4\theta,\xi^6\theta) \le$

$$\kappa(|d(\theta,\xi^2\theta) - d(\xi^4\theta,\xi^6\theta)| + d(\theta,\xi^2\theta) + |d(\xi^2\theta,\xi^4\theta) - d(\xi^2\theta,\xi^4\theta)|) + \Lambda\ min\{d(\theta,\xi^2\theta), d(\xi^2\theta,\xi^4\theta)\}$$

(2.16) $d(\xi^4\theta,\xi^6\theta) \le \kappa(|d(\theta,\xi^2\theta) - d(\xi^4\theta,\xi^6\theta)| + d(\theta,\xi^2\theta))$

$$+\Lambda\ min\{d(\theta,\xi^2\theta), d(\xi^2\theta,\xi^4\theta)\}$$

Case-(i):

(2.17) $d(\theta,\xi^2\theta) \le d(\xi^4\theta,\xi^6\theta)$

(2.18) $d(\xi^4\theta, \xi^6\theta) \le \frac{\Lambda}{(1-\kappa)} d(\theta,\xi^2\theta) < d(\theta,\xi^2\theta)$ $(\because \Lambda + \kappa < 1)$

If $d(\theta,\xi^2\theta) \ne 0$, then $d(\xi^4\theta,\xi^6\theta) < d(\theta,\xi^2\theta) \le d(\xi^4\theta,\xi^6\theta)$
which is a contradiction that $d(\theta,\xi^2\theta) \ne 0$.
Therefore $d(\theta,\xi^2\theta) = 0 \Rightarrow \xi^2\theta = \theta$

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Therefore θ is a fixed point of ξ^2.

Case-(ii): $d(\theta, \xi^2\theta) > d(\xi^4\theta, \xi^6\theta)$
Therefore $d(\xi^4\theta, \xi^6\theta) \leq \frac{2\kappa+\Lambda}{1+\kappa} d(\theta, \xi^2\theta)$ $\quad (\because\ \kappa + \Lambda < 1, \frac{2\kappa+\Lambda}{1+\kappa} < 1)$.
Therefore
(2.19) $\quad d(\xi^4\theta, \xi^6\theta) \leq \mu d(\theta, \xi^2\theta) where\ \ \mu = \frac{2\kappa+\Lambda}{1+\kappa}$
In general, we can show that
(2.20) $\quad d(\xi^{2n}\theta, \xi^{2n+2}\theta) \leq \mu^n d(\theta, \xi^2\theta) \to 0\ as\ n \to \infty$
However, taking into account the sequence $\{\theta_n\}_{n\in N}$ defined as follows
$\theta_1 = \xi^2\theta_0,\ \theta_2 = \xi^4\theta_0, \dots, \theta_n = \xi^{2n}\theta_0$, where $\theta_0 \in \Theta$.
From Equation 2.6, $d(\theta_n, \theta_{n+1}) \leq cd(\theta_{n-1}, \theta_n)$, for $n \in N$.
Therefore from lemma1.2, we get that $\{\theta_n\}_{n\in N}$ forms a Cauchy sequence in the complete b-metric space Θ. As a result, it is convergent, and there is $u \in \Theta$ and $\ni \lim_{n\to\infty} d(\theta_n, u) = 0$.
Since ξ^2 is continuous mapping, then
$\lim_{n\to\infty} d(\theta_n, \xi^2 u) = \lim_{n\to\infty} d(\xi^{n-1}\theta_{n-1}, \xi^2 u)$=0 and we conclude that $\xi^2 u = u$, i.e., u is a fixed point of ξ^2.
Suppose θ is a fixed point of ξ^2, ($\xi^2\theta = \theta$) and replace ψ by $\xi\theta$ in Equation (2.12).
(2.21) $\quad \alpha(\theta, \xi\theta)d(\theta, \xi\theta) \leq \kappa(|d(\theta, \xi\theta) - d(\xi^2\theta, \xi^3\theta)| + d(\theta, \xi\theta) + |d(\xi\theta, \xi^2\theta) - d(\xi\theta, \xi^2\theta)|) + \Lambda min\{d(\theta, \xi\theta), d(\xi\theta, \xi^2\theta)\}$
$\alpha(\theta, \xi\theta)d(\theta, \xi\theta) \leq \kappa(|d(\theta, \xi\theta) - d(\xi^2\theta, \xi^3\theta)| + d(\theta, \xi\theta) + \Lambda min\{d(\theta, \xi\theta), d(\xi\theta, \xi^2\theta)\})$

$$\alpha(\theta, \xi\theta)d(\theta, \xi\theta) \leq (\kappa + \Lambda)d(\theta, \xi\theta)$$

$(\alpha(\theta, \xi\theta) - (\kappa + \Lambda)d(\theta, \xi\theta))d(\theta, \xi\theta) \leq 0$ either $\alpha(\theta, \xi\theta) \leq \kappa + \Lambda$ or $\xi\theta = \theta$ i.e., θ is a fixed point of ξ.

Example 2.1. For $\Theta = [0,2]$, let $d: \Theta \times \Theta \to [0, \infty)$ be a standard metric, i.e., $d(\theta, \psi) = |\theta - \psi|$.
Let $\alpha: \Theta \times \Theta \to [0, \infty)$ be a function, defined by

$$\alpha(\theta, \psi) = \begin{cases} 1 & for\ \ \theta, \psi \in [0,2] \\ 0 & \text{otherwise} \end{cases}$$

In terms of Θ, a self-mapping ξ is defined as

$$\xi(\theta) = \begin{cases} 1 + \theta, & if\ \theta \in [0,1) \\ \theta - 1, & if\ \theta \in [1,2] \end{cases}$$

Then

$$\xi^2\theta = \begin{cases} \theta, & if\ \theta \in [0,1) \\ \theta, & if\ \theta \in [1,2] \end{cases}$$

ξ is α-orbital admissible, and as an illustration,
$\alpha(\theta, \xi\theta) \geqslant 1 \Rightarrow \alpha(\xi\theta, \xi^2\theta) \geqslant 1$.
Clearly $\xi^2\theta$ is continuous on $[0, 2]$, but ξ is not continuous at $\theta = 1$ and ξ is Type-I IStratescu α-almost Contraction, the mapping ξ^2 has a fixed point, using Theorem 2.2.

Example 2.2. Let $\Theta = [0,1]$, and the function $d: \Theta \times \Theta \to [0, \infty)$, that is $d(\theta, \psi) = |\theta - \psi|$.
Let the mapping $\alpha: \Theta \times \Theta \to [0, \infty)$ defined as

$$\alpha(\theta, \psi) = \begin{cases} 1 & for\ \ \theta, \psi \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

In terms of Θ, a self-mapping ξ is defined as

$$\xi(\theta) = \begin{cases} \frac{1}{2}, & if\ \theta \in [0,1) \\ \frac{1}{4}, & if\ \theta = 1 \end{cases}$$

Then

$$\xi^2\theta = \begin{cases} \frac{1}{2}, & if\ \theta \in [0,1) \\ \frac{1}{2}, & if\ \theta = 1 \end{cases}$$

we have that the mapping ξ^2 is continuous, but ξ is not.
For example, $\alpha(1, \xi 1) = \alpha(\xi 1, 1) = \alpha(1, \frac{1}{4}) \geqslant 1$ where ξ is an α −orbital admissible.
For $\theta, \psi \in [0, 1]$, we have $d(\xi^2\theta, \xi^2\psi) = d(\frac{1}{2}, \frac{1}{2}) = 0$.

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So clearly, ξ is Type-I Istratescu α-almost Contraction.
For $\theta, \psi \in [0, 1)$ and $\psi = \xi^2\theta$

$$➼\to^{*}(\theta, \psi) = |d(\theta, \xi\theta) - d(\xi\psi, \xi^2\psi)| + d(\theta, \psi) + |d(\psi, \xi\psi) - d(\xi\theta, \xi^2\theta)|$$

$$= |d(\theta, \tfrac{1}{2}) - d(\xi^3\theta, \xi^4\theta)| + d(\theta, \xi^2\theta) + |d(\xi^2\theta, \xi^3\theta) - d(\xi\theta, \xi^2\theta)|$$

References

1. Istratescu, V. Some fixed point theorems for convex contraction mappings and mappings with convex diminishing diameters (I). Ann. Mat. Pura Appl. 1982, 130, 89-104.
2. Istratescu, V. Some fixed point theorems for convex contraction mappings and mappings with convex diminishing diameters (II). Ann. Mat. Pura Appl. 1983, 134, 327-362.
3. Berinde, V. Approximating fixed points of weak contractions using the picard iteration, Nonlinear Anal. Forum 2004, 9, 43-53.
4. Hussain, N. Doric, D. Kadelburg, Z. Radenovic, S., Suzuki-type fixed point results in metric type spaces, Fixed Point Theory Appl., 2012, 126.
5. Aydi, H. Bota, M. F., Karapinar, F., Moradi, S.,A Common fixed point for weak ϕ-contractions on b-metric spaces, Fixed Point Theory, 2012, 13, 337-346.
6. Bota, M.F., Karapinar, F., Mlesnite, O., Ulam-Ayers stability results for fixed point problems via alpha -psi-contractive mapping in b-metric space, Abst. Appl. Anal., 2013, 825293.
7. Bota, M.F., Karapinar, E., A note on a some results on a some results on multi-valed weakly lungek mappings in b-metric space,Cent. Eur. J. Math., 2013, 11, 1711-1712.
8. Bota, M. Chifa, C. Karapinar, E. Fixed point theorems for generalized $(\alpha - \phi)$-Ciric-type contractive multivalued operators in b-metric spaces, J. Nonlinear Sci. Appl. 2016, 9, 1165- 1177.
9. Khan, M.S. Singh, Y.M. Maniu, G. Postolache, M. On generalized convex contractions of type-2 in b-metric and 2-metric spaces. J. Nonlinear Sci. Appl. 2017, 10, 2902-2913.
10. Miculesu, R. Mihail, A. New fixed point theorems for set-valued contractions in b-metric spaces, J. Fixed Point Theory Appl. 2017, 19, 2153-2163.
11. Popescu, O. Some new fixed point theorems for α-Geraghty-Contraction type maps in metric spaces, Fixed Point Theory Appl., 2014, 190.