Journal of Chemical Health Risks
www.jchr.org
JCHR (2023) 13(4), 1718-1725 | ISSN:2251-6727

Existence of fixed point in neutrosophic fuzzy metric spaces via common property (E.A.) with application

Sunakshi Kapoor [1], Vishal Gupta[2,*] and Naveen Gulati[3]

[1,2]Department of Mathematics, MMDU, Mullana (Haryana), India
[3]Department of Mathematics, SD College, Ambala Cantt (Haryana), India

* Correspondence:

(Received: 02 September 2023 Revised: 14 October Accepted: 07 November)

KEYWORDS

NFMS, compatible mappings, common property (E.A.), implicit relation

ABSTRACT: In this present work, we employ common property (E.A.) on NFMS with the help of implicit functions in order to prove FPT. The notion of compatible mappings is also used. This paper generalizes the concept of FPT by using E.A. property on FMS. An application is also given in support of our result.

Introduction: Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Vitae sapien pellentesque habitant morbi tristique senectus et netus. Dignissim cras tincidunt lobortis feugiat vivamus at augue eget arcu. At risus viverra adipiscing at in. Cras semper auctor neque vitae tempus quam. Sed cras ornare arcu dui. Turpis massa sed elementum tempus. Risus commodo viverra maecenas accumsan lacus vel facilisis volutpat est. Dictum non consectetur a erat nam at. Lorem mollis aliquam ut porttitor leo. Egestas sed sed risus pretium quam.

Objectives: Lacinia at quis risus sed vulputate odio ut enim. Orci porta non pulvinar neque laoreet suspendisse interdum. Consequat mauris nunc congue nisi vitae suscipit. Morbi quis commodo odio aenean.

Methods: At varius vel pharetra vel turpis nunc eget lorem. Feugiat scelerisque varius morbi enim nunc. Cras semper auctor neque vitae tempus quam pellentesque nec. Faucibus purus in massa tempor nec feugiat nisl. Congue nisi vitae suscipit tellus mauris a. Est sit amet facilisis magna etiam tempor. Dictum varius duis at consectetur. Purus semper eget duis at tellus at urna. Ipsum consequat nisl vel pretium. Viverra maecenas accumsan lacus vel facilisis volutpat est. Bibendum arcu vitae elementum curabitur vitae nunc sed. Nisl tincidunt eget nullam non nisi est. Ac turpis egestas integer eget aliquet nibh praesent. Auctor augue mauris augue neque gravida in fermentum et sollicitudin. Turpis egestas integer eget aliquet nibh praesent tristique magna. Libero justo laoreet sit amet cursus sit amet dictum.

Results: Egestas diam in arcu cursus euismod quis viverra nibh. Convallis aenean et tortor at risus viverra. Sit amet justo donec enim diam. Sem et tortor consequat id. Purus gravida quis blandit turpis. Consectetur adipiscing elit duis tristique sollicitudin nibh sit amet commodo. Eget duis at tellus at urna condimentum mattis pellentesque. Auctor elit sed vulputate mi sit amet. Consequat ac felis donec et. In dictum non consectetur a erat nam at lectus. Dui vivamus arcu felis bibendum ut tristique. Lacinia quis vel eros donec ac. Ac turpis egestas maecenas pharetra convallis posuere morbi leo. Tortor id aliquet lectus proin.

Conclusions: Mi tempus imperdiet nulla malesuada. Magna fermentum iaculis eu non diam phasellus vestibulum. Consectetur adipiscing elit duis tristique sollicitudin nibh sit amet commodo. Elit scelerisque

Journal of Chemical Health Risks
www.jchr.org
JCHR (2023) 13(4), 1718-1725 | ISSN:2251-6727

mauris pellentesque pulvinar. Et malesuada fames ac turpis egestas maecenas pharetra convallis posuere. Elementum integer enim neque volutpat ac tincidunt vitae semper.

1. Introduction

Anonymous problems in our daily life deal with uncertainties. While dealing with crisp set we are not able to solve these problems. Zadeh **[1]** was considered as the Lord of fuzziness who started it in 1965. The growing interest in the branch of FS lead to the origination of further branches of FMS proposed by Kramosil and Michalek **[2]** in 1975 and George and Veeramani **[3]** in 1994. Then Park in 2004 gave the concept of IFMS. Then the concept of NFMS was given by M. Jeyaramen in 2021. Contractions play a great role in proving FPT for different branches of FMS. In the present paper the influence of Common Property (E.A.) on NFMS is considered in the presence of Implicit functions and then Branciari-Integral equation is being solved by using suitable contraction which is being highlighted as an application of this paper.

2. Preliminaries

Definition 2.1. [2] "An operator $\circ$: $[0, 1] \times [0, 1] \to [0, 1]$ is called continuous t-norm if:

1. $\circ$ is commutative and associative;
2. $\circ$ is continuous;
3. $e \circ 1 = e$, for all $e \in [0, 1]$;
4. $e \circ d \leq f \circ h$, whenever it is given that $e \leq f$ and $d \leq h$ for all $e, d, f, h \in [0,1]$."

Definition 2.2.[2] "An operator $\triangle$: $[0, 1] \times [0, 1] \to [0, 1]$ is taken as a continuous t-co-norm if:

1. $\triangle$ is commutative and associative;
2. $\triangle$is continuous;
3. $e \triangle 0 = e$, for all $e \in [0, 1]$;
4. $e \triangle d \leq f \triangle h$, whenever it is given that $e \leq f$ and $d \leq h$ for all $e, d, f, h \in [0, 1]$."

Definition 2.3. ([3]) "A three-tuple $(U, L, \circ)$ is considered as FMS, where $\circ$ is t-norm and L is FS defined on $U^2 \times [0, \infty)$, which obeys:-

1. $L(w, d, 0) > 0$;
2. $L(w, d, f) = 0$ iff $w = d$, where $f > 0$;
3. $L(w, d, f) = L(d, w, f)$;
4. $L(w, d, f) \circ L(d, r, s) \leq L(w, r, f + s)$, wherew, $d, r \in U$ and $s, f >$
5. $L(w, d, \cdot) : [0, \infty) \to [0, 1]$ being left continuous;
6. $\lim_{f \to \infty} L(w, d, f) = 1$."

Definition 2.4. ([4]) **"**A 5-tuple $(U, E, D, \circ, \triangle)$ is considered as IFMS if U is any arbitrary set,$\circ$ is considered as a t-norm, $\triangle$ is considered as a t-co-norm. E andD are FS on $U \times U \times [0, \infty)$, then for all $e, d, f \in U$ and $b, c > 0$:-

1. $E(e, d, b) + D(e, d, b) \leq 1$;
2. $E(e, d, 0) = 0$;
3. $E(e, d, b) = 1$ for all $b > 0$ if and only if $e = d$;
4. $E(e, d, b) = E(d, e, b)$;
5. $E(e, d, b) = 1$ as $b \to \infty$;
6. $E(e, d, b) \circ E(d, f, c) \leq E(e, f, b + c)$;
7. $E(e, d, \cdot) : [0, \infty) \to [0, 1]$ is left continuous;
8. $D(e, d, 0) = 1$;
9. $D(e, d, b) = 0$ for all $b > 0$ if and only if $e = d$;
10. $D(e, d, b) = D(d, e, b)$;
11. $D(e, d, b) = 0$ as $b \to \infty$;
12. $D(e, d, b) \triangle D(d, f, c) \geq D(e, f, b + c)$;
13. $D(e, d, \cdot) : [0, \infty) \to [0, 1]$ is right continuous.

Here, $E(e, d, b)$ and $D(e, d, b)$ depict closeness and non-closeness in between e and d with respect to b, c respectively."

Definition 2.5.([4]) Neutrosophic fuzzy metric space (NFMS): "The four-tuple set $(F, N, \circ, \triangle)$ with F as a universal set, $\circ$ as a t-norm, $\triangle$ as a t-co-norm is called NFMS when the following conditions are satisfied for all $w, q, r \in F$,

1. $0 \le G(w, q, \lambda) \le 1$, $0 \le B(w, q, \lambda) \le 1$, $0 \le Y(w, q, \lambda) \le 1$, for all $\lambda \in R^+$;
2. $G(w, q, \lambda) + B(w, q, \lambda) + Y(w, q, \lambda) \le 3$, for $\lambda \in R^+$;
3. $G(w, q, \lambda) = 1$ if and only if $w = q$, for $\lambda > 0$;
4. $G(w, q, \lambda) = G(q, w, \lambda)$, for $\lambda \in R^+$;
5. $G(w, q, \lambda) \circ G(q, r, £) \le G(w, r, \lambda + £)$ $(\lambda, £ \in R^+)$;
6. $G(w, q, \cdot) : [0, \infty) \to [0, 1]$ is continuous;
7. $\lim_{\lambda \to \infty} G(w, q, \lambda) = 1$ $(\lambda \in R^+)$;
8. $B(w, q, \lambda) = 0$ if and only if $w = q$, for $\lambda \in R^+$;
9. $B(w, q, \lambda) = B(q, w, \lambda)$ (for $\lambda \in R^+$);
10. $B(w, q, \lambda) \triangle B(q, r, £) \ge B(w, r, \lambda + £)$ $(\lambda, £ \in R^+)$;
11. $B(w, q, \cdot) : [0, \infty) \to [0, 1]$ is continuous;
12. $\lim_{\lambda \to \infty} B(w, q, \lambda) = 0$ (for $\lambda \in R^+$);
13. $Y(w, q, \lambda) = 0$ if and only if $w = q$, for $\lambda \in R^+$;
14. $Y(w, q, \lambda) = Y(q, w, \lambda)$ (for $\lambda \in R^+$);
15. $Y(w, q, \lambda) \triangle Y(q, r, £) \ge Y(w, r, \lambda + £)$ $(\lambda, £ \in R^+)$;
16. $Y(w, q, \cdot) : [0, \infty) \to [0, 1]$ is continuous;
17. $\lim_{\lambda \to \infty} Y(w, q, \lambda) = 0$, for $\lambda > 0$;
18. If $\lambda \le 0$ then $G(w, q, \lambda) = 0$, $B(w, q, \lambda) = 1$ and $Y(w, q, \lambda) = 1$.

Then $(F, N, \circ, \triangle)$ is called NMS on F. The functions $G(w, q, \lambda)$, $B(w, q, \lambda)$, $Y(w, q, \lambda)$ denote the degree of nearness, the degree of neutralness and the degree of non-nearness between w, q, r of N with respect to λ, respectively."

Then M. Jeyaraman and S. Sonndrarajan [6] in 2021 gave the definition of NFMS in a different way.

Definition 2.6. [6] "A 6-tuple $(T, L, Q, E, \circ, \triangle)$ is taken as NFMS if T is any arbitrary non-empty set, $\circ$ being continuous t-norm, $\triangle$ being continuous t-co-norm. L, Q and E are Neutrosophic sets defined on $T^3 \times IR$ satisfying the under given conditions for all $b, c, d, \delta \in T$:

1. $0 \le L(b, c, d, \delta) \le 1$, $0 \le Q(b, c, d, \delta) \le 1$, $0 \le E(b, c, d, \delta) \le 1$;
2. $L(b, c, d, \delta) + Q(b, c, d, \delta) + E(b, c, d, \delta) \le 3$;
3. $L(b, c, d, \delta) = 1$ iff $b = c = d$;
4. $L(b, c, d, \delta) = L(t(b, c, d, \delta))$, where t is the permutation function;
5. $L(b, c, d, \delta) \circ L(r, z, z, \vartheta) \le L(b, c, z, \delta + \vartheta)$, for all $\delta, \vartheta > 0$;
6. $L(b, c, d, \cdot): [0, \infty) \to [0, 1]$ is Neutrosophic continuous;
7. $\lim_{\delta \to \infty} L(b, c, d, \delta) = 1$, for all $\delta > 0$;
8. $Q(b, c, d, \delta) = 0$ iff $b = c = d$;
9. $Q(b, c, d, \delta) = Q(t(b, c, d, \delta))$, where t is the permutation function;
10. $Q(b, c, d, \delta) \triangle Q(r, z, z, \vartheta) \ge Q(b, c, z, \delta + \vartheta)$, for all $\delta, \vartheta > 0$;
11. $Q(b, c, d, \cdot): [0, \infty) \to [0, 1]$ is Neutrosophic continuous;
12. $\lim_{\delta \to \infty} Q(b, c, d, \delta) = 0$, for all $\delta > 0$;
13. $E(b, c, d, \delta) = 0$ iff $b = c = d$;
14. $E(b, c, d, \delta) = E(p(b, c, d, \delta))$, where p is the permutation function;
15. $E(b, c, d, \delta) \triangle E(r, z, z, \vartheta) \ge E(b, c, z, \delta + \vartheta)$, for all $\delta, \vartheta > 0$;
16. $E(b, c, d, \cdot): [0, \infty) \to [0, 1]$ is Neutrosophic continuous;
17. $\lim_{\delta \to \infty} E(b, c, d, \delta) = 0$, for all $\delta > 0$;
18. If $\delta >$ 0 then $L(b, c, d, \delta) = 0$, $Q(b, c, d, \delta) = 1$, $E(b, c, d, \delta) = 0$.

Then $(T, L, Q, E, °, \Delta)$ is called a NMS on T. The functions L, Q, E describes the degree of closedness, neutralness and non-closedness between e, d and r with respect to δ respectively."

Definition 2.7. [6] In NFMS $(T, L, Q, E, \circ, \triangle)$, $L(b, c, d, \cdot)$ is non-decreasing, $Q(b, c, d, \cdot)$ and $E(b, c, d, \cdot)$ are non-increasing for all $b, c, d \in T$.

Definition 2.8. Mappings (F, S) which are self-mappings of NFMS $(T, L, Q, E, \circ, \triangle)$ exhibit the E.A.property if we can find a sequence $\{u_n\}$ in T such that $\lim_{n\to\infty} F u_n = \lim_{n\to\infty} S u_n = z$, for some $z \in T$.

Definition 2.9. Pairs (A, S) and (B, F) from T to T of a NFMS $(T, L, Q, E, \circ, \triangle)$ possess the E.A. property if sequences $\{e_n\}$ and $\{d_n\}$ in T can be found such that $\lim_{n\to\infty} Ae_n = \lim_{n\to\infty} Se_n = \lim_{n\to\infty} Bd_n = \lim_{n\to\infty} Fd_n = z$, for some $z \in T$.

Journal of Chemical Health Risks
www.jchr.org
JCHR (2023) 13(4), 1718-1725 | ISSN:2251-6727

Definition 2.10. Two mappings A and B which are considered to be self-maps of an NFMS$(T, L, Q, E, \circ, \triangle)$ are taken as compatible iff $\lim_{n\to\infty} L(ABw_n, BAw_n, f) \to 1$, where $f > 0$ and $\{w_n\}$ being a sequence in T satisfying $ABw_n, Bw_n \to i$, for any i in T as $n \to\infty$.

Definition 2.11. Mappings A and B which are taken to be self-maps are taken as owcm iff there exists a point i in T at which they both commute.

Definition 2.12. Maps (A, B) which are self-maps of an NFMS$(T, L, Q, E, \circ, \triangle)$ are considered as SC if $\lim_{n\to\infty} ABw_n = Bw$, whenever $\{w_n\}$ is a sequence satisfying $\lim_{n\to\infty} Aw_n = \lim_{n\to\infty} Bw_n = i$, for a few i in T. It describes that (A, B) is SC and $Ar = Br$ then $ABr = BAr$.

Definition 2.13.A combination of self-mappings (F, S) of a NFMS$(T, L, Q, E, \circ, \triangle)$ is taken as a WC if they coincide only at some point that is $Fu = Su$, for some $u \in T$. Then $FS\,u = SF\,u$.

Definition 2.14. [7] "Various types of real continuous functions $\rho, \theta: [0, 2]^4 \to IR$ which represent implicit relations are:-

1. $\theta (c , 1 , c, 1) \geq 0$ this implies$c \geq 1$;
2. $\theta (c , 1, 1, c) \geq 0$ this implies $c \geq 1$;
3. $\theta (c , c, 1, 1) \geq 0$ this implies$c \geq 1$;
4. $\rho (c , 0, c, 0) \leq 0$ this implies$c \leq 0$;
5. $\rho (c , 0, 0, c) \leq 0$ this implies$c \leq 0$;
6. $\rho (c , 0, c, 0) \leq 0$, for all $c \geq 0$ this implies$c \leq 0$."

Definition 2.15. [8] "(Branciari-Integral Contractive type Condition)

Let (T, d) be a Complete MS, $c \in (0, 1)$ and let f be a mapping such that for every $w, q \in T$,

$\int_0^{d(f(w), f(q))} f(t)\, dt \leq \int_0^{d(w, q)} f(t)\, dt$holds, where

$g: [0, \infty) \to [0, \infty)$ is a Lebesgue-Integrable self-mapping which own the property of summability on every subset of T which is compact and which shows positiveness, then for each$\in > 0$, f has a unique fixed point $a \in T$, such that for each $e \in T$, $\lim_{n\to\infty} f_n(e) = a$."

Lemma 2.16. Let A_i, S and F are self-maps of a NFMS$(T, L, Q, E, \circ, \triangle)$ satisfying the following:-

1. (A_0, F) possess the property (E.A.).
2. for any $e, d \in T$and ρ , θbeing implicit relations and for all $t > 0$, a positive number $k \in (0, 1)$ can be found such that,

$$\theta\left(\left(L(A_1e, A_od, kt), L(Se, Fd, t), L(Se, A_1e, t), L(Fd, A_od, t)\right) \geq 0 \qquad (2.1)$$

$$\rho\left(\left(Q(A_1e, A_od, kt), Q(Se, Fd, t), Q(Se, A_1e, t), Q(Fd, A_od, t)\right) \leq 0 \qquad (2.2)$$

$$A(T) \subseteq F(T) \text{ or } A_o(T) \subseteq S(T).$$

Then the combination (A_1, S) and (A_0, F) exhibit the common (E.A.) property.

Proof. Since (A_o, F)posses the common property E.A. therefore, there exists $\{u_n\} \subset T$ such that $\lim_{n\to\infty} A_ou_n = \lim_{n\to\infty} fu_n = m$for some $m \in T$.Also since $A_o \subset S$, so, there exists some $\{v_n\} \in T$such that $A_ou_n = Sv_n$for each v_n.Hence, $\lim_{n\to\infty} A_ou_n = \lim_{n\to\infty} Sv_n = m$, for some $m \in T$.So, we have $\lim_{n\to\infty} A_ou_n = \lim_{n\to\infty} fu_n = \lim_{n\to\infty} Sv_n = m$.
Now, $\lim_{n\to\infty} A_1v_n = m$ iff $\lim_{n\to\infty} L(A_ou_n, A_1v_n, t) = 1$.
Assume that $\lim_{n\to\infty} A_1v_n \neq m$, then there exists a subsequence $\{A_1v_{nk}\} \subset \{A_1v_n\}$such that $\lim_{n\to\infty} L(A_ou_n, A_1v_{nk}, t) = y$.Then from (2) the conditions of implicit functions are not satisfied hence

$$\lim_{n\to\infty} A_1v_n = m.$$

Hence, the pairs $(A_o, F)$and (A_1, S)exhibit the common property (E.A.)

3. Main Results

Journal of Chemical Health Risks
www.jchr.org
JCHR (2023) 13(4), 1718-1725 | ISSN:2251-6727

Theorem 3.1.Let A_i, S and U are self-mappings of a NFMS (T, L, Q, E, ∘, △) satisfying and the following conditions:-

1. $[(A_1, S)$ and $(A_0, F)]$ exhibit the common property (E.A.);
2. S(T), F(T) and U(T) are closed subsets of T.

Then the combinations $[(A_1, S), (A_0, F)]$ and $[(A_1, S), (A_2, F)]$ provide points of Coincidence. Moreover A_i, S, F and U have unique common fixed point assuming that both the combination $[(A_1, S), (A_0, F)]$ and $[(A_1, S), (A_2, F)]$ are WC.

Proof. Suppose there exist two sequences $\{p_n\}$ and $\{d_n\}$ in T such that

$$\lim_{n\to\infty} A_o d_n = \lim_{n\to\infty} A_1 p_n = \lim_{n\to\infty} Sp_n = \lim_{n\to\infty} Fd_n$$

for some $n \in T$. Since $\delta(e)$is a subset of T which is closed. We can find $u \in T$ for which $n = Su$. We claim that $A_1u = n$. If $A_1u \neq n$ then take $p = u, d = d_n$ in (2.1) and (2.2)

$\theta\big(L(A_1u, A_od_n, kt), L(Su, Fd_n, t), L(Su, A_1u, t), L(Fd_n, A_od_n, t)\big) \geq 0;$

On taking $n\to\infty$, we get

$\theta(L(A_1u, n, kt), L(n, n, t), L(n, A_1u, t), L(n, n, t)) \geq 0;$

so, $\theta(L(A_1u, n, kt), 1, L(n, A_1u, t), 1) \geq 0.$

and $\rho(Q(A_1u, A_od_n, kt), Q(S_u, Fd_n, t), Q(Su, A_1u, t), Q(Fd_n, A_od_n, t)) \leq 0;$

On taking $n\to\infty$, we get

$\rho(Q(A_1u, n, kt), Q(n, n, t), Q(n, A_1u, t), Q(n, n, t)) \leq 0;$

so, $\rho(Q(A_1u, n, kt), 0, Q(n, A_1u, t), 0) \leq 0.$

As θ and ρ are increasing, we have $\theta(L(A_1u, n, t), 1, L(n, A_1u, t), 1) \geq 0$

and $\rho(Q(A_1u, n, kt), 0, Q(n, A_1u, t), 0) \geq 0.$

$L(A_1u, n, t) \geq 1$ and $L(A_1u, n, t) \leq 0.$

$L(A_1u, n, t) = 1$ and$Q(A_1u, n, t)= 0.$

$A_1u = n= Su$, which implies u is the point of coincidence for (A_1, S).

Since F(T)is a closed subset of T. $Fd_n= n$ inF(T). Hence we can discover $v \in T$ for which $Fv = n = A_1u = Su$. Now we will show that $A_0u = n$. If not then take $e = u, d = v$, we have

$\theta\big(L(A_1u, A_ov, kt), L(Su, Fv, t), L(Su, A_1u, t), L(Fv, A_ov, t)\big) \geq 0;$

so, $\theta\big(L(n, A_ov, kt), 1, 1, L(n, A_ov, t)\big) \geq 0.$

and $\rho\big(Q(A_1u, A_ov, kt),Q(Su, Fv, t), Q(Su, A_ou, t),Q(Fv, A_ov,t)\big) \leq 0;$

so, $\rho\big(Q(n, A_ov, kt), 0, 0, Q(n, A_ov, t)\big) \leq 0.$

As θ and ρ are non-decreasing in the first case, we get

$$\theta\big(L(n, A_ov, kt), 1, 1, L(n, A_ov, t)\big) \geq 0,$$

and $\rho\big(Q(n, A_ov, kt), 0, 0,Q(n, A_ov, t)\big) \leq 0.$

so, $L(n, A_ov, kt) \geq 1$ and $Q(n, A_ov, kt) \leq 0.$

Hence, $L(n, A_ov, kt) = 1$ and $Q(n, A_ov, kt) = 0.$

$A_0v = n = Fv$, which enhance the fact thatv is the point of coincidence of (A_o, F).Since the combinations(A_1, S) and (A_o, F) are WC and $A_1u = Su, A_ov = Fv$. So,

$$A_1n = A_1Su = SA_1u = Sn = A_oFv = FA_gv = Fn$$

If $A_1v \neq n$ then we have,

$\theta\big(L(A_1n, A_0v, kt), L(Sn, Fv, t), L(Sn, A_1n, t), L(Fv, A_0v, t)\big) \geq 0;$

and

$\theta(L(A_1n, n, kt), L(Sn, Fv, t), L(Sn, A_1n, t), L(Fv, A_0v, t)) \geq 0;$

and $\theta(L(A_1n, n, kt), L(A_1n, n, t), L(A_1n, A_1n, t), L(n, n, t)) \geq 0;$

Journal of Chemical Health Risks
www.jchr.org
JCHR (2023) 13(4), 1718-1725 | ISSN:2251-6727

Hence

$\theta(L(A_1 n, n, kt), L(A_1 n, n, t), 1, 1)) \geq 0.$

and $\rho(Q(A_1 n, A_0 v, kt), Q(Sn, Fv, t), Q(Sn, A_1 n, t), Q(Fv, A_0 v, t)) \leq 0;$

so,

$\rho(Q(A_1 n, n, kt), Q(A_1 n, n, t), Q(A_1 n, A_1 n, t), Q(n, n, t)) \leq 0;$

$\rho(Q(A_1 n, n, kt), Q(A_1 n, n, t), 0, 0)) \leq 0.$

As θ and ρ are non-decreasing in the first argument, we have

$\theta(L(A_1 n, n, t), L(A_1 n, n, t), 1, 1) \geq 0;$

and

$\rho(Q(A_1 n, n, kt), Q(A_1 n, n, t), 0, 0) \leq 0.$

$L(A_1 n, n, t) \geq 1$ and $Q(A_1 n, n, t) \leq 0.$

$L(A_1 n, n, t) = 1$ and $Q(A_1 n, n, t) = 0.$

$A_1 n = n = Sn$. Similarly, $A_0 n = Fn = n$.Hence $A_0 n = A_1 n = Sn = Fn$, which impliesn is their common fixed point

Similarly, same steps can be taken for (A_1, S) and (A_2, U) by considering $\{p_n\}$and $\{n_n\}$as sequence inT and by taking the same implicit functions ρ and θ.

Uniqueness

Letn andw be two common fixed points of A_1, A_0, S and F. If $n \neq w$ then we have,

$\theta(L(A_1 n, A_d w, kt), L(Sn, Fw, t), L(Sn, A_1 n, t), L(Fw, A_o w, t)) \geq 0;$

$\theta(L(n, w, kt), L(n, w, t), L(n, n, t), L(w, w, t)) \geq 0;$

$\theta(L(n, w, t), L(n, w, t), 1, 1) \geq 0.$

and $\rho(Q(A_1 n, A_0 w, kt), Q(Sn, Fw, t), Q(Sn, A_1 n, t), Q(Fw, A_0 w, t)) \leq 0;$

Hence,

$\rho(Q(n, w, kt), Q(n, w, t), Q(n, n, t), Q(w, w, t)) \leq 0;$

$\rho(Q(n, w, t), Q(n, w, t), 0, 0) \leq 0$, which implies $L(n,w, t) \geq 1$ and $Q(n, w, t) \leq 0$.

This gives, $L(n, w, t) = 1$ and $Q(n, w, t) = 0$.

Thus

$n = w$

Similarly, let n and u be two fixed points for A_1, A_2, S and U. Then on the similar note we can prove that

$n = u$

From (3.1) and (3.2) n = w = u.

This gives the existence for their unique common fixed point.

4. Applications

Theorem 4.1. Let $(T, L, Q, E, \circ, \triangle)$ be a NFMS. Let E, Q, R, S, W, F be maps from T to T such that

1. $E(T) \subseteq S(T)$, $Q(T) \subseteq R(T)$, $E(T) \subseteq W(T)$, $F(T) \subseteq R(T)$.
2. [(E, R), (Q, S)] and[(E, R), (F, W)]satisfy E. A. property.
3. $Q(A_1 b, A_o d, gt), Q(Sb, Fd, t), Q(Sb, A_1 b, t), Q(Fd, A_o d, t) \leq 0.$
4. We can findg ∈(0, 1)such that for everye, d, r ∈ T, t > 0, for all λ > 0
$\int_0^{L(Eb,Qd,gt)} \rho(t)dt \geq \int_0^{U(b,d,t)} \rho(t)dt;$
$\int_0^{Q(Eb,Qd,gt)} \rho(t)\, dt \leq \int_0^{V(b,d,t)} \rho(t)\, dt$, where$\rho: IR^+ \to IR$ is Lebesgue-Integrable map and

$U(b, d, t) = L(Rb, Sd, t) \circ L(Eb, Re, t) \circ L(Qd, Sd, t) \circ L(Eb, Sd, t)$

$V(b, d, t) = Q(Rb, Sd, t) \triangle Q(Eb, Re, t) \triangle Q(Qd, Sd, t) \triangle Q(Eb, Sd,t).$

If out of E(b), Q(b), R(b), S(b) any one is a complete subspace of T, then (E, R)and(Q, S) will own a

Journal of Chemical Health Risks
www.jchr.org
JCHR (2023) 13(4), 1718-1725 | ISSN:2251-6727

coincidence point. Further if (E, R) and (Q, S) are taken to be WC, then E, Q, R and S will exhibit a common fixed point in T which will be unique.

Proof. Let (Q, S) own the $E.A.$property. Then we can search a sequence $\{b_n\}$ in T, for which $\lim_{n\to\infty} Qb_n = \lim_{n\to\infty} Sb_n = q$, where $u \in T$. Also $Q(T) \subseteq R(T)$ then there exists $\{d_n\}$ in T such that $Qb_n = Rd_n$. We get $R(d_n) = q$ as $n \to\infty$. Now, we will show that $\lim_{n\to\infty} Ed_n = q$. To enhance it, put $b = d_n$, $d = b_n$ in condition (3). We get, $\int_0^{L(Ed_n,Qb_n,gt)} \rho(t)dt \geq \int_0^{U(d_n,b_n,t)} \rho(t)dt$,

and $\int_0^{Q(Ed_n,Qb_n,gt)} \rho(t)dt \leq \int_0^{V(d_n,b_n,t)} \rho(t)dt$.

$U(d_n, b_n, t) = L(Rd_n, Sb_n, t) \circ L(Ed_n, Rd_n, t) \circ L(Qb_n, Sb_n, t) \circ L(Ed_n, Sb_n, t)$,

and $V(d_n, b_n, t) = Q(Rd_n, Sb_n, t) \triangle Q(Ed_n, Rd_n, t) \triangle Q(Qb_n, Sb_n, t) \triangle Q(Ed_n, Sb_n, t)$.

From the above results $\lim_{n\to\infty} Ed_n = q = \lim_{n\to\infty} Sd_n$. Let S(b) is a subspace of T which is complete. Then q = S (m) for some $m \in E$.

$\lim_{n\to\infty} Ed_n = q = \lim_{n\to\infty} Rd_n = \lim_{n\to\infty} Qb_n = \lim_{n\to\infty} Sb_n$.

Now we will prove that, $E(m) = R(m)$. Taking $e = m$, $d = b_n$ in (3), by the impact of above equation, we have

$$\int_0^{L(Em,Qb_n,gt)} \rho(t)dt \geq \int_0^{U(m,b_n,t)} \rho(t)dt$$

and $\int_0^{Q(Em,Qb_n,gt)} \rho(t)dt \leq \int_0^{V(m,b_n,t)} \rho(t)dt$,

where $U(m, b_n, t) = L(Rm, Sb_n, t) \circ L(Em, Rmt) \circ L(Qb_n, Sb_n, t) \circ L(Em, Sb_n, t)$,

$V(m, b_n, t) = Q(Rm, Sb_n, t) \triangle Q(Em, Rm, t) \triangle Q(Qb_n, Sb_n, t) \triangle Q(Em, Sb_n, t)$.

As $n \to\infty$ then we get $E(m) = R(m)$. This supports the fact that (E, R) have coincident point $m \in T$, the WC of (E, R) implies that $E R(m) = RE(m)$. Thus $E E(m) = ER(m) = RE(m) = RR(m)$.

As $E(b) \subset S(b)$, there exists $E \in T$ such that $E(m) = S(p)$. Next, we claim that $S(p) = Q(p)$. Taking $b = m$, $d = p$, we get

$$\int_0^{L(Em,Qp,gt)} \rho(t)dt \geq \int_0^{U(m,p,t)} \rho(t)dt$$

and $\int_0^{Q(Em,Qp,gt)} \rho(t)dt \leq \int_0^{V(m,p,t)} \rho(t)dt$,

where, $U(m, p, t) = L(Rm, Sp, t) \circ L(Em, Rm, t) \circ L(Qp, Sp, t) \circ L(Em, Sp, t)$,

$V(m, p, t) = Q(Rm, Sp, t) \triangle Q(Em, Rm, t) \triangle Q(Qp, Sp, t) \triangle Q(Em, Sp, t)$.

By using above results, we get $Em = Qp$,

$$Em = Rm = Sp = Qp.$$

The WC of (Q, S) implies that QSp, SQp. Thus $QSp = SQp = QQp = SSp$

Next we prove that Em is the common fixed point of E, Q, R and S. Take $b = Em$, $d = p$

$$\int_0^{L(Em,Qb_n,kt)} \rho(t)\, dt \geq \int_0^{U(m,b_n,t)} \rho(t)dt$$

and $\int_0^{L(Em,Qb_n,kt)} \rho(t)\, dt \geq \int_0^{U(m,b_n,t)} \rho(t)dt$,

where, $(Em, p, t) = L(Rm, Sp, t) \circ L(EEm, REm, t) \circ L(Qp, Sp, t) \circ L(EEm, Sp, t)$,

$V(Em, p, t) = Q(Rbm, Sp, t) \triangle Q(EEm, Rem, t) \triangle Q(Qp, Sp, t) \triangle Q(EEm, Sp, t)$.

$Em = EEm = REm$ is common fixed point of E and R. Similarly Qp is fixed point of S and Q which is common. Since $Em = Qp$. Hence Em is the fixed point of E, Q, R and S. Similarly, Em can be proved as the fixed point of E, R, F, W. Hence we can easily enhance the uniqueness of fixed point by using the above result.

5. Conclusions

In this chapter the authors have studied the impact of E.A. property on NFMS in the presence of implicit functions and invariant point results have been proved for this consideration.

Funding: This research has received no external funding.

Acknowledgments: I would like to thank my Ph.D. guide Dr. Vishal Gupta for his valuable guidance and immense support. We would like to thank all the university personnels who helped us to achieve our goal,our almighty who created such appropriate environment for our research.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. L.A. Zadeh, Fuzzy sets. Information and Control, (1965), 8(3), 338-353.
2. I. Kramosil, J. Michaleg, Fuzzy Metric and Statistical Metric Spaces, Kybernetica, 1975, 11(5), 326-334.
3. A. George, P. Veeramani. On some results in fuzzy metric spaces. Fuzzy Sets and Systems, 1997, 90(3), 336-344.
4. D. Turgoglu, C. Alaca, C. Yildin. Compatible maps and Compatible maps of type (α) and type (β) in intuitionistic fuzzy metric spaces. Demonstration Math, 2006, 39(3), 671-684.
5. Murat KIRISCI, Necip SIMISEK. Neutrosophic Soft Sets with Medical Decision-Making applications. Sigma J Eng and Nat Sci, 2019, 10(2), 231-235.
6. S Sowndrarajan, M. Jeyaraman, Florentin Samarandache. Fixed Point Results for Contractive Theorems in Neutrosophic Metric Space. Neutrosophic Sets and Systems, 36(2020).
7. V. Gupta, A. Kanwar.Fixed point theorem in fuzzy metric spaces satisfying E.A. property. Indian J. Sci Tech, 2012, 5(12), 3767-3769.
8. Branciari. A fixed point theorem for mappings satisfying a general contractive condition of integral type. Int. J. Math, Sci. Tech , 2002, 29(6), 531-536.
9. V. Gupta, N. Mani. Existence and Uniqueness of fixed point in fuzzy metric spaces and its applications. Advances in Intelligent Systems and Computing, Springer, 2014, vol. 236, 217-224.
10. V. Gupta, N. Mani.Fixed point theorems using control function in fuzzy metric spaces. cogent Math. 2015, 2(1), Article 1D1053173.
11. C. Alaca, I. Altun, D. Turgoglu. On Compatible maps of type (α) and type (β) in IFM- spaces. Common Korean Math, Soc. 2006, 39(3), 427-446.

Received: Month Day, Year. Accepted: Month Day, Year